Application of Antenna Arrays to Mobile Communications, Part II: Beam-Forming and Direction-of-Arrival Considerations

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Array processing involves manipulation of signals induced on various antenna elements. Its capabilities of steering nulls to reduce cochannel interferences and pointing independent beams toward various mobiles, as well as its ability to provide estimates of directions of radiating sources, make it attractive to a mobile communications system designer. Array processing is expected to play an important role in fulfilling the increased demands of various mobile communications services. Part I of this paper showed how an array could be utilized in different configurations to improve the performance of mobile communications systems, with references to various studies where feasibility of an array system for mobile communications is considered.

This paper provides a comprehensive and detailed treatment of different beam-forming schemes, adaptive algorithms to adjust the required weighting on antennas, direction-of-arrival estimation methods—including their performance comparison—and effects of errors on the performance of an array system, as well as schemes to alleviate them. This paper brings together almost all aspects of array signal processing. It is presented at a level appropriate to nonexperts in the field and contains a large reference list to probe further.

Keywords— Beam forming, conjugate gradient method, eigenstructure methods, ESPRIT, least square algorithm, linear prediction method, maximum entropy, maximum likelihood method, minimum norm, mobile communications, multipath arrivals, MUSIC, MVDR estimator, neural networks, recursive least square algorithm, weighted subspace fitting.

NOMENCLATURE

A	L by M matrix, with its columns being the steering vectors.
A_i	Amplitude of the ith source using frequency modulation.
$\underline{a}_k$	Denotes $L-1$ weights after the kth tap in TDL structure in a beam-space processor.
AIC	Akaike's information criterion.
B	Blocking matrix or the matrix prefilter for a narrow-band beam-space processor.
BER	Bit error rate.
BPSK	Binary phase shift keying.
C	$LJ \times J$ constraint matrix.
c	Speed of propagation of a plane wave front.
CANAL	Concurrent nulling and location.
CDMA	Code division multiple access.
CMA	Constant modulus algorithm.
CRLB	Cramer–Rao lower bound.
d	Interelement spacing of a linear equispaced array.
$d_i(n)$	Message symbol in TDMA system and message sequence in CDMA system (associated with the ith source).
DOA	Direction of arrival.
$E[\cdot]$	Expectation operator.
$\underline{e}_1$	Vector of all zeros except the first element, which is equal to unity.
ESPRIT	Estimation of signal parameters via rotational invariance technique.
$\underline{F}$	J-dimensional vector specifying the frequency response in the look direction.
f_0	Center frequency.
f_N	Nyquist frequency.
FBW	Fractional bandwidth.
FDMA	Frequency division multiple access.
FFT	Fast Fourier transform.
FINE	First principal vector.
FIR	Finite impulse response.
$\hat{G}$	Array gain of the optimal processor.
$G_{xy}(f)$	Cross-power spectrum of two broad-band signals $x(t)$ and $y(t)$.
$g(t)$	Pseudo-random noise binary sequence having the values $+1$ or -1.
$\underline{g}(\underline{w}(n))$	Unbiased estimate of the gradient of the mean squared error or the mean output power.
GMSK	Gaussian minimum shift keying.
GSC	Generalized side-lobe canceller.
GSM	Global system for mobile communications.
$H(f)$	Transfer function.

HEOS	Highly elliptical orbit satellite.
I	Identity matrix.
J	Number of taps in a tapped delay line structure.
J_0	Reflection matrix with all its elements along the secondary diagonal being equal to unity and zero elsewhere.
$J(n)$	Cumulative mean square error at the nth iteration, cost function.
K	Number of elements in a subarray.
$k_{\underline{w}\underline{w}}(n)$	Covariance matrix of the weights at the nth iteration.
L	Number of elements in an array.
$\ell n[\cdot]$	Natural logarithm of $[\cdot]$.
LMS	Least mean square.
LS	Least square.
M	Number of directional sources, number of beams in a beam-space processor.
M	Misadjustment.
$m_i(t)$	Complex modulating function of the ith source.
$m_s(n)$	Modulating function of the signal at time instant n.
$m_s(t)$	Modulating function of the signal source at time instant t.
MAP	Maximum *a posteriori*.
MDL	Minimum description length.
MEM	Maximum entropy method.
min-norm	Minimum norm.
ML	Maximum likelihood.
MLM	Maximum likelihood method.
MMSE	Minimum mean squared error.
MSE	Mean squared error.
MVDR	Minimum variance distortionless response.
MUSIC	Multiple signal classification.
N	Number of samples.
N_i	Number of possible combinations of elements with lag i.
$n_\ell(t)$	Random noise component on the ℓth element.
NAME	Noise-alone matrix inverse.
P	Projection operator.
$P_B(\theta)$	Power estimated by Barrette method as a function of θ.
$P_{\mathrm{LP}}(\theta)$	Power estimated by linear prediction method as a function of θ.
$P_{\mathrm{ME}}(\theta)$	Power estimated by MEM as a function of θ.
$P_{\mathrm{MN}}(\theta)$	Power estimated by minimum norm method as a function of θ.
$P_{\mathrm{MU}}(\theta)$	Power estimated by MUSIC method as a function of θ.
$P_{\mathrm{MV}}(\theta)$	Power estimated by MVDR method as a function of θ.
P_N	Output noise power.
$P(\underline{w})$	Mean output power of the processor for a given $\underline{w}$.
p_i	Power of the ith source as measured at the reference element.
p_I	Power of a directional interference.
p_s	Power of the source in the look direction, referred to as the signal source.
$P(\underline{w}_c)$	Mean output power of the conventional processor.
$p(t)$	Sampling pulse.
PIC	Postbeam-former interference canceller.
QPSK	Quadrature phase shift keying.
$\underline{q}(t)$	$M-1$ dimensional vector denoting outputs of $M-1$ auxiliary beams of a beam-space processor.
R	Array correlation matrix.
$R(n)$	Array correlation matrix estimate at time instant n.
R_m	mth subarray matrix of the forward method.
$\bar{R}_m$	mth subarray matrix of the backward method.
R_N	Noise-only array correlation matrix.
$r(t)$	Reference signal.
r_i	ith correlation lag.
$\underline{r}_\ell$	Position vector of the ℓth element.
$\underline{r}_d$	LJ-dimensional vector denoting correlation between the desired signal and the array signal vector.
RLS	Recursive least square.
RMS	Root mean square.
$\underline{s}_0$	Steering vector in the look direction.
$\underline{s}_I$	Steering vector in direction (ϕ_I, θ_I).
$\underline{s}_i$	Steering vector associated with the direction (ϕ_i, θ_i) or the ith source.
$\underline{s}_\theta$	Steering vector associated with the direction θ.
S	M by M matrix denoting the source correlation.
$S(f)$	Power density of broad-band signal $s(t)$.
SMI	Sample matrix inversion.
SNR	Signal-to-noise ratio.
SPNMI	Signal-plus-noise matrix inverse.
STD	Standard deviation.
T	Delay between successive taps of TDL filter.
T_0	Bulk delay.
$T_\ell(\phi_0, \theta_0)$	Steering delay in front of ℓth element to steer an array in (ϕ_0, θ_0) direction.
T_ℓ	Steering delay in front of ℓth element to steer an array in look direction.
TAM	Toeplitz approximation method.
TDL	Tap delay line.
TDMA	Time division multiple access.
TLS	Total least square.
$\mathrm{Tr}(R)$	Trace of R.
U_N	Matrix with its L–M columns being the eigenvectors corresponding to the L–M smallest eigenvalues of R.
U_S	Matrix with its M columns being the

	eigenvectors corresponding to the M largest eigenvalues.
$\underline{U}_\ell$	Unit-norm eigenvector corresponding to λ_ℓ.
$\underline{u}_1$	Column vector of all zeros except one element, which is equal to one.
ULA	Uniformly spaced linear array.
$\underline{V}(n)$	Difference between estimated weights and the optimal weights at the nth iteration.
$V_g(\underline{w}(n))$	Variance of the gradient.
$\hat{\underline{v}}(\phi_i, \theta_i)$	Unit vector in direction (ϕ_i, θ_i).
W_s	Matrix prefilter to block the look direction in a broad-band beam-space processor.
w_ℓ^*	Weighting on ℓth element for the narrow-band beam former.
$\underline{w}$	Array weight vector.
$\bar{\underline{w}}$	Mean of the estimated weights at the nth iteration.
$\underline{w}(n+1)$	Array weight vector at time instant $n+1$, new weights computed at the $(n+1)$th iteration
$\underline{w}_c$	Array weights of the conventional beam former.
$\underline{w}_m$	L weights after the $(m-1)$th tap in TDL structure.
$\hat{\underline{w}}$	Weights of the optimal beam former.
$\hat{\underline{w}}_{\text{MSE}}$	Weights with minimum mean squared error.
WSF	Weighted subspace fitting.
x_ℓ	Total signal induced on the ℓth element due to all M directional sources and background noise.
$x_{\ell s}(t)$	Signal induced on the ℓth element due to the signal sources only.
$\underline{x}(n)$	Array signal vector at time instant n.
$\underline{x}(t)$	Array signal vector at time instant t.
$\underline{x}'(t)$	$L-1$ dimensional signal vector following matrix prefilter.
$\underline{x}_s(t)$	Array signal vector at time instant t due to the signal sources only.
$\underline{x}_N(n)$	Array receiver vector not containing the signal at time instant n.
$y(t)$	Output of a beam former at time t.
$y(n)$	Output of a beam former when it is operating with weights $\underline{w}(n)$.
$\tilde{y}(n)$	Modified output of a beam former when it is operating with weights $\underline{w}(n)$.
$y_c(t)$	Output of the main beam of a beam-space processor.
$y_a(t)$	Weighted output of the auxiliary beams of a beam-space processor.
y_0	Desired amplitude in the absence of interference.
$\underline{z}$	Correlation between the reference signal and the array signals vector.
$\hat{\alpha}$	Output SNR of the optimal processor.
Δ	Sampling interval.
Δ_0	Magnitude of the displacement vector.
δ_0	Forgetting factor.
δ	Complex scalar denoting the correlation between the signal and an interference.
$\delta_{xy}(f)$	Correlation between two broad-band signals $x(t)$ and $y(t)$.
$\varepsilon(\cdot)$	Error signal.
$\tilde{\varepsilon}(n)$	Error signal between the reference signal and modified output.
$\Delta\tilde{\varepsilon}(n)$	Change in error signal when array output is perturbed by a small amount Δy.
$\varepsilon(\underline{w}(n))$	Error between the array output and the reference signal for a given $\underline{w}(n)$.
(ϕ_0, θ_0)	Look direction.
(ϕ_I, θ_I)	Direction of an interference.
(ϕ_i, θ_i)	Direction of the ith source.
$\xi_i(t)$	Message part of the ith source using frequency modulation.
Λ	L by L diagonal matrix with λ_ℓ, $\ell = 1$, L being its diagonal entries.
λ_ℓ	ℓth eigenvalue of the array correlation matrix.
$\lambda_{\max}$	Maximum eigenvalue of R.
$\lambda_\ell(PRP)$	ℓth eigenvalue of PRP.
$\lambda_{\max}(PRP)$	Maximum eigenvalue of PRP.
μ_0	Constant.
μ	Gradient step size.
$\mu(n)$	Step size at the nth iteration.
ρ	Scalar quantity, which depends on the direction of the interference relative to the signal source and the array geometry.
$\rho(\tau)$	Correlation function of a broad-band signal.
$\rho_{xy}(\tau)$	Cross-correlation function.
Σ	L by L matrix with $\underline{U}_\ell$, $\ell = 1$, L being its columns.
σ_n^2	Variance of random noise.
σ_q^2	Variance of quantization noise.
(θ_i)	Direction of the ith source.
$\hat{\tau}_\ell$	Time constant of the ℓth trajectory.
$\tau_\ell(\theta_i)$	Time taken by a plane wave arriving from the ith source in direction (θ_i) and measured from the ℓth element to the reference point.
$\tau_\ell(\phi_i, \theta_i)$	Time taken by a plane wave arriving from the ith source in direction (ϕ_i, θ_i) and measured from the ℓth element.
$\tau_{ij}(\theta)$	Differential delay between elements i and j due to a source in direction θ.
$*$	Complex conjugate.
$(\ )^T$	Transpose of a vector or matrix.
$(\ )^H$	Complex conjugate transpose of a vector or matrix.

I. INTRODUCTION

The demand for wireless mobile communications services is growing at an explosive rate, with the anticipation

that communication to a mobile device anywhere on the globe at all times will be available in the near future. An array of antennas mounted on vehicles, ships, aircraft, satellites, and base stations is expected to play an important role in fulfilling these services' increased demand for channels and in realizing the dream that a portable communication device the size of a wristwatch will be available at an affordable cost for such services.

Part I of this paper showed how an antenna array could be used in various configurations to improve the performance of mobile communications systems, with references to theoretical analyses, computer simulations, and experimental system developments.

Array signal processing involves the manipulation of signals induced on the elements of an array. The widespread interest in the subject area has been maintained over decades due to its applicability to many walks of life. The first issue of IEEE TRANSACTIONS ON ANTENNAS AND PROPAGATION, published in 1964 [1], has been followed by many special issues of various journals [2]–[6], a number of books [7]–[12], a selected bibliography [13], and a vast amount of specialized research papers. Some of the general papers that discuss various issues include [14]–[31].

This paper provides a comprehensive review of various beam-forming schemes, adaptive algorithms to adjust the required weighting on antennas, DOA estimation methods, and array-system sensitivity to parameter perturbations. As array signal processing has applications in many other disciplines, this paper aims to provide a complete treatment of the subject area by extending coverage to topics that might not be directly relevant to mobile communications. This paper, however, provides references where beam-forming and DOA estimation methods have been suggested for mobile communications systems.

In Section II, a signal model useful for array processing is presented along with various beam-forming schemes, including descriptions of conventional delay and sum beam formers, null steering, constrained beam forming and optimization using a reference signal, beam-space processing, broad-band array processing in time and frequency domains, digital beam forming, and eigenstructure methods. Section III describes adaptive algorithms to adjust the weights of an array. These algorithms include SMI, unconstrained as well as constrained LMS, normalized LMS, structured gradient, RLS, CMA, conjugate gradient method, and neural-network approach to beam forming. Some discussion on implementation issues, convergence characteristics of adaptive algorithms, and signal sensitivity of the LMS algorithm is also provided in this section.

Section IV describes various DOA estimation methods, compares their performance, and analyzes their sensitivity. These methods include spectral estimation, MVDR estimator, linear prediction, maximum entropy, ML, various eigenstructure methods—including many versions of MUSIC algorithms—min-norm, CLOSEST, ESPRIT, and WSF. This section also contains a discussion on various preprocessing and number-of-source estimation methods.

Section V discusses the effect of errors and perturbations on the performance of the array processing schemes. A signal model applicable to multipath situations is discussed, and it is pointed out how multipath degrades the performance of an array processing system. Various cures for multipath degradation are highlighted in this section, which also presents a discussion on look direction and steering vector errors, element failure and element position errors, and weight errors. References to many robust beam-forming schemes are also included in this section. Section VI concludes this paper.

II. BEAM FORMING

In this section, various beam-forming methods are discussed in detail. First, notation, terminology, and a signal model useful for this purpose are introduced.

A. Terminology and Signal Model

Consider an array of L omnidirectional elements immersed in a homogeneous media in the far field of M uncorrelated sinusoidal point sources of frequency f_0. Let the origin of the coordinate system be taken as the time reference, as shown in Fig. 1. Thus, the time taken by a plane wave arriving from the ith source in direction (ϕ_i, θ_i) and measured from the ℓth element to the origin is given by

$$\tau_\ell(\phi_i, \theta_i) = \frac{\underline{r}_\ell \cdot \hat{\underline{v}}(\phi_i, \theta_i)}{c} \tag{1}$$

where $\underline{r}_\ell$ is the position vector of the ℓth element, $\hat{\underline{v}}(\phi_i, \theta_i)$ is the unit vector in direction (ϕ_i, θ_i), c is the speed of propagation of the plane wave front, and $\cdot$ represents the inner product. For a linear array of equispaced elements with element spacing d aligned with the x-axis such that the first element is situated at the origin, it becomes

$$\tau_\ell(\theta_i) = \frac{d}{c}(\ell - 1)\cos\theta_i. \tag{2}$$

The signal induced on the reference element due to the ith source is normally expressed in complex notation as

$$m_i(t)e^{j2\pi f_0 t} \tag{3}$$

with $m_i(t)$ denoting the complex modulating function. The structure of the modulating function reflects the particular modulation used in a communications system. For example, for an FDMA system, it is a frequency-modulated signal given by $m_i(t) = A_i e^{j\xi_i(t)}$, with A_i denoting the amplitude and $\xi_i(t)$ denoting the message. For a TDMA system, it is given by

$$m_i(t) = \sum_n d_i(n)p(t - n\Delta) \tag{4}$$

where $p(t)$ is the sampling pulse, the amplitude $d_i(n)$ denotes the message symbol, and Δ is the sampling interval. For a CDMA system, $m_i(t)$ is given by
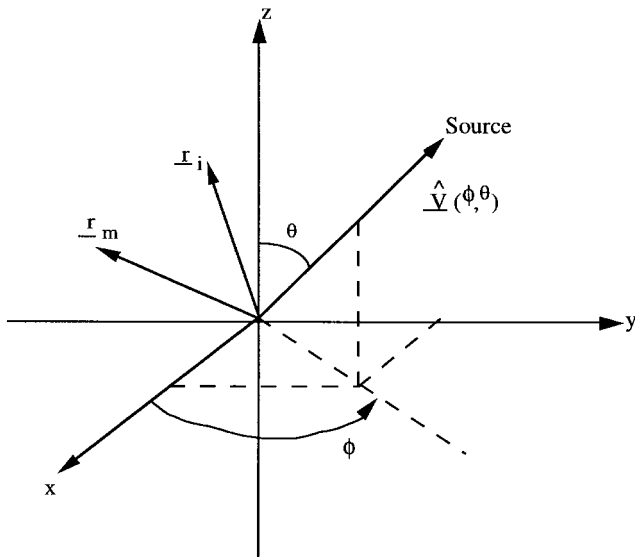
$$m_i(t) = d_i(t)g(t) \tag{5}$$

Fig. 1. Definition of coordinate system.

Fig. 2. Narrow-band beam-former structure.

where $d_i(n)$ denotes the message sequence and $g(t)$ is a pseudo-random noise binary sequence having the values $+1$ or -1 [32].

In general, the modulating function is normally modeled as a complex low-pass process with zero mean and variance equal to the source power p_i, as measured at the reference element.

Assuming that the wavefront on the ℓth elements arrives $\tau_\ell(\phi_i, \theta_i)$ seconds before it arrives at the reference element, the signal induced on the ℓth element due to the ith source can be expressed as

$$m_i(t)e^{j2\pi f_0(t+\tau_\ell(\phi_i,\theta_i))}. \qquad (6)$$

The expression is based upon the narrow-band assumption for array signal processing, which assumes that the bandwidth of the signal is narrow enough and that the array dimensions are small enough for the modulating function to stay almost constant during $\tau_\ell(\phi_i, \theta_i)$ seconds, that is, the approximation $m_i(t) \cong m_i(t + \tau_\ell(\phi_i, \theta_i))$ holds.

Let x_ℓ denote the total signal induced due to all M directional sources and background noise on the ℓth element. Then it is given by

$$x_\ell = \sum_{i=1}^{M} m_i(t)e^{j2\pi f_0(t+\tau_\ell(\phi_i,\theta_i))} + n_\ell(t) \qquad (7)$$

where $n_\ell(t)$ is a random noise component on the ℓth element, which includes background noise and electronic noise generated in the ℓth channel. It is assumed to be temporally white with zero mean and variance equal to σ_n^2.

It should be noted that if the elements were not omnidirectional, then the signal induced on each element due to a source is scaled by an amount equal to the response of the element under consideration in the direction of the source.

Consider a narrow-band beam former, shown in Fig. 2, where signals from each element are multiplied by a complex weight and summed to form the array output. The figure does not show components such as preamplifiers,
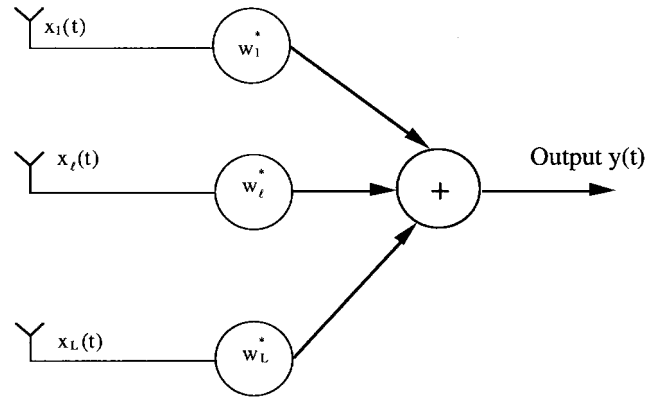
bandpass filters, and so on. It follows from the figure that an expression for the array output is given by

$$y(t) = \sum_{\ell=1}^{L} w_\ell^* x_\ell(t) \qquad (8)$$

where $*$ denotes the complex conjugate.

Denoting the weights of the beam former as

$$\underline{w} = [w_1, w_2, \cdots, w_L]^T \qquad (9)$$

and signals induced on all elements as

$$\underline{x}(t) = [x_1(t), x_2(t), \cdots, x_L(t)]^T \qquad (10)$$

the output of the beam former becomes

$$y(t) = \underline{w}^H \underline{x}(t) \qquad (11)$$

where superscripts T and H, respectively, denote the transpose and complex conjugate transpose of a vector or matrix. Throughout this paper, $\underline{w}$ and $\underline{x}(t)$ are referred to as the array weight vector and the array signal vector, respectively.

If the components of $\underline{x}(t)$ can be modeled as zero mean stationary processes, then for a given $\underline{w}$, the mean output power of the processor is given by

$$\begin{aligned} P(\underline{w}) &= E[y(t) \quad y^*(t)] \\ &= \underline{w}^H R \underline{w} \end{aligned} \qquad (12)$$

where $E[\cdot]$ denotes the expectation operator and R is the array correlation matrix defined by

$$R = E[\underline{x}(t) \quad \underline{x}^H(t)]. \qquad (13)$$

Elements of this matrix denote the correlation between various elements. For example, R_{ij} denotes the correlation between the ith and the jth element of the array. Denote the steering vector associated with the direction (ϕ_i, θ_i) or the ith source by an L-dimensional complex vector $\underline{s}_i$ as

$$\underline{s}_i = [\exp(j2\pi f_0\tau_1(\phi_i,\theta_i)), \cdots, \exp(j2\pi f_0\tau_L(\phi_i,\theta_i))]^T. \qquad (14)$$

Algebraic manipulation using (7), (10), and (13) leads to the following expression for R

$$R = \sum_{i=1}^{M} p_i \underline{s}_i \underline{s}_i^H + \sigma_n^2 I \qquad (15)$$

where I is an identity matrix and p_i denotes the power of the ith source measured at one of the elements of the array. It should be noted that p_i is the variance of the complex modulating function $m_i(t)$ when it is modeled as a zero mean low-pass random process, as mentioned previously.

Using matrix notation, the correlation matrix R may be expressed in the following compact form:

$$R = ASA^H + \sigma_n^2 I \tag{16}$$

where columns of the L by M matrix A are made up of steering vectors, i.e.,

$$A = [\underline{s}_1, \underline{s}_2, \cdots, \underline{s}_M] \tag{17}$$

and M by M matrix S denotes the source correlation. For uncorrelated sources, it is a diagonal matrix with

$$S_{ij} = \begin{cases} p_i, & i = j \\ 0, & i \neq j. \end{cases} \tag{18}$$

Sometimes, it is useful to express R in terms of its eigenvalues and their associated eigenvectors. The eigenvalues of R can be divided into two sets when the environment consists of uncorrelated directional sources and uncorrelated white noise.

The eigenvalues contained in one set are of equal values. Their value does not depend upon the directional sources and is equal to the variance of the white noise. The eigenvalues contained in the second set are a function of the parameters of the directional sources, and their number is equal to the number of these sources. Each eigenvalue of this set is associated with a directional source, and its value changes with the change in the source power of this source. The eigenvalues of this set are bigger than those associated with white noise. Sometimes, these eigenvalues are referred to as the signal eigenvalues, and the others belonging to the first set are referred to as the noise eigenvalues. Thus, the R of an array of L elements immersed in M directional sources and the white noise has M signal eigenvalues and L–M noise eigenvalues.

Denoting the L eigenvalues of R in descending order by λ_ℓ, $\ell = 1, L$ and their corresponding unit-norm eigenvectors by $\underline{U}_\ell$, $\ell = 1, L$, the matrix takes the following form:

$$R = \Sigma \Lambda \Sigma^H \tag{19}$$

with a diagonal matrix

$$\Lambda = \begin{bmatrix} \lambda_1 & & & \\ & \cdot & & 0 \\ & & \lambda_\ell & \\ & 0 & & \cdot \\ & & & \lambda_L \end{bmatrix} \tag{20}$$

and

$$\Sigma = [\underline{U}_1 \quad \cdots \quad \underline{U}_L]. \tag{21}$$

This representation sometimes is referred to as the spectral decomposition of R. Using the fact that the eigenvectors form an orthonormal set, this leads to the following expression for R

$$R = \sum_{\ell=1}^{M} \lambda_\ell \underline{U}_\ell \underline{U}_\ell^H + \sigma_n^2 I. \tag{22}$$

There are many schemes to select the weights of the beam former depicted in Fig. 2, each with its own characteristics and limitations. Some of these are now discussed.

B. Conventional Beam Former

A conventional beam former is a simple beam former, sometimes known as the delay-and-sum beam former, with all its weights of equal magnitudes. The phases are selected to steer the array in a particular direction (ϕ_0, θ_0), known as the look direction. With $\underline{s}_0$ denoting the steering vector in the look direction, the array weights are given by

$$\underline{w}_c = \frac{1}{L} \underline{s}_0. \tag{23}$$

The array with these weights has unity response in the look direction, that is, the mean output power of the processor due to a source in the look direction is the same as the source power. This may be understood as follows.

Assume that there is a source of power p_s in the look direction, hereafter referred to as the signal source, with $m_s(t)$ denoting its modulating function. The signal induced on the ℓth element due to this source only is given by

$$x_{\ell s}(t) = m_s(t) e^{j2\pi f_0 (t + \tau_\ell(\phi_0, \theta_0))}. \tag{24}$$

Thus, in vector notation, using a steering vector to denote relevant phases, the array signal vector due to the look direction signal becomes

$$\underline{x}_s(t) = m_s(t) e^{j2\pi f_0 t} \underline{s}_0 \tag{25}$$

and the output of the array with weight vector $\underline{w}_c$ becomes

$$\begin{aligned} y(t) &= \underline{w}_c^H \underline{x}_s(t) \\ &= m_s(t) e^{j2\pi f_0 t} \end{aligned} \tag{26}$$

yielding the mean output power of the processor

$$\begin{aligned} P(\underline{w}_c) &= E[y(t) \quad y^*(t)] \\ &= p_s. \end{aligned} \tag{27}$$

Thus, the mean output power of the conventional beam former steered in the look direction is equal to the power of the source in the look direction. The process is similar to steering the array mechanically in the look direction except that it is done electronically by adjusting the phases. This is also referred to as electronic steering, and phase shifters are used to adjust the required phases. It should be noted that the aperture of an electronically steered array is different from that of a mechanically steered array.

The concept of a delay-and-sum beam former can be further understood with Fig. 3, which shows an array with two elements separated by distance d. Assume that a plane wave arriving from direction θ induces voltage $s(t)$ on the
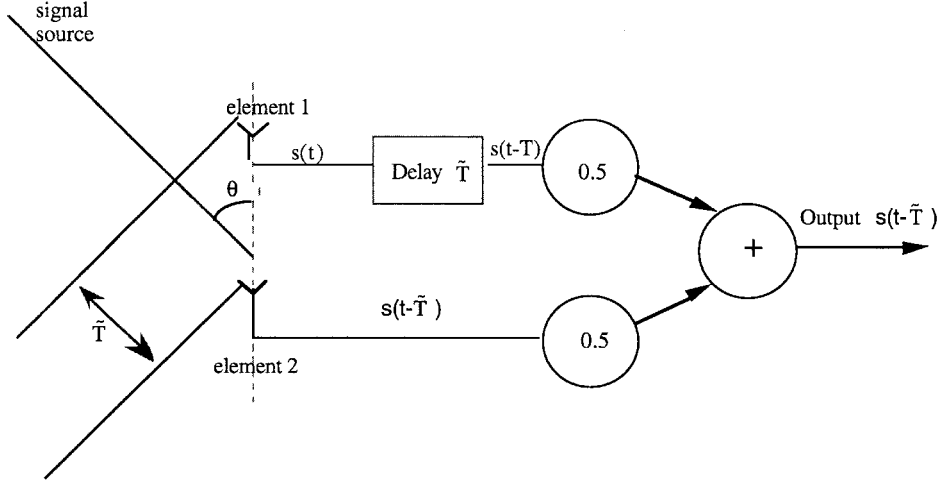
Fig. 3. Two-element delay-and-sum beam-former structure.

first element. As the wave arrives at the second element $\tilde{T}$ seconds later, with

$$\tilde{T} = \frac{d}{c}\cos\theta \qquad (28)$$

the induced voltage on the second element equals $s(t - \tilde{T})$. If the voltage induced at the first element is delayed by an amount equal to $\tilde{T}$, producing voltage $s(t - \tilde{T})$, and no delay is provided at the second element, then both voltage waveforms appear in phase and the output of the beam former is produced by summing these waveforms. A scaling of each waveform by 0.5 provides the gain in direction θ equal to unity.

In an environment consisting of only uncorrelated noise and no directional interferences, this beam former provides maximum SNR. For uncorrelated noise, the R_N is given by

$$R_N = \sigma_n^2 I \qquad (29)$$

and the output noise power of the beam former

$$
\begin{aligned}
P_N &= \underline{w}_c^H R_N \underline{w}_c \\
&= \frac{\sigma_n^2}{L}.
\end{aligned} \qquad (30)
$$

It shows that the noise power at the array output is L times less than that present on each element. Thus, the processor with unity gain in the signal direction has reduced the uncorrelated noise by L, yielding the output SNR $= p_s L / \sigma_n^2$. As the input SNR is p_s / σ_n^2 this provides an array gain, which is defined as the ratio of the output SNR to the input SNR, equal to L the number of elements in the array.

Though this beam former provides maximum output SNR when there is no directional jammer operating at the same frequency, it is not effective in the presence of directional jammers, intentional or unintentional. The response of the processor toward a source in direction (ϕ_I, θ_I) is given by

$$\underline{w}_c^H \underline{s}_I = \frac{1}{L}\underline{s}_0^H \underline{s}_I \qquad (31)$$

where $\underline{s}_I$ denotes the steering vector in direction (ϕ_I, θ_I). In the next section, a beam former that puts nulls in the directions of interferences is described.

C. Null-Steering Beam Former

A null-steering beam former is used to cancel a plane wave arriving from a known direction and thus produces a null in the response pattern in the DOA of the plane wave. One of the earliest schemes, referred to as DICANNE [33], [34], achieves this by estimating the signal arriving from a known direction by steering a conventional beam in the direction of the source and then subtracting the output of this from each element. An estimate of the signal is made by delay-and-sum beam forming using shift registers to provide the required delay at each element such that the signal arriving from the beam-steering direction appears in phase after the delay. Then these waveforms are summed with equal weighting. This signal is then subtracted from each element after the delay. The process is very effective for canceling strong interference and could be repeated for multiple interference cancellation.

Though the process of subtracting the estimated interference using a delay-and-sum beam former used by DICANNE scheme is easy to implement for single interference, it becomes cumbersome as the number of interferences grows. A beam with unity response in the desired direction and nulls in interference directions may be formed by estimating the weights of a beam former, shown in Fig. 2, using suitable constraints [22], [34]. Assume that $\underline{s}_0$ is the steering vector in the direction where unity response is required and that $\underline{s}_1, \cdots, \underline{s}_k$ are k steering vectors associated with k directions where nulls are required. The desired weight vector is the solution of following simultaneous equations:

$$\underline{w}^H \underline{s}_0 = 1 \qquad (32)$$

$$\underline{w}^H \underline{s}_i = 0, \qquad i = 1, \cdots, k. \qquad (33)$$

Using matrix notation, this becomes

$$\underline{w}^H A = \underline{c}_1^T \qquad (34)$$

where A is a matrix with its columns being the steering vectors associated with all directional sources, including the look direction, that is

$$A \triangleq [\underline{s}_0, \underline{s}_1, \cdots, \underline{s}_k] \qquad (35)$$

and $\underline{e}_1$ is a vector of all zeros except the first element, which is one, that is

$$\underline{e}_1 = [1, 0, \cdots, 0]^T. \qquad (36)$$

For $k = L - 1$, A is a square matrix. Assuming that the inverse of A exists, which requires that all steering vectors are linearly independent [35], the solution for the weight vector is given by

$$\underline{w}^H = \underline{e}_1^T A^{-1}. \qquad (37)$$

In case the steering vectors are not linearly independent, A is not invertible, and its pseudo inverse can be used in its place.

It follows from this equation that due to the structure of the vector $\underline{e}_1$, the first row of the inverse of matrix A forms the weight vector. Thus, the weights selected as the first row of the inverse of matrix A have the desired properties of unity response in the look direction and nulls in the directions of interferences.

When the number of required nulls is less than $L - 1$, A is not a square matrix. A suitable estimate of weights may be produced using

$$\underline{w}^H = \underline{e}_1^T A^H (A A^H)^{-1}. \qquad (38)$$

Though the beam pattern produced by this beam former has nulls in the directions of interferences, it is not designed to minimize the uncorrelated noise at the array output. It is possible to achieve this by selecting weights that minimize the mean output power subject to the above constraints [36].

An application of a null-steering scheme for detecting an amplitude modulated signal by placing nulls in the known directions of interferences is described in [37], which is able to cancel a strong jammer in a mobile communications system. The use of a null-steering scheme for a transmitting array employed at a base station, discussed in [38], minimizes the interferences toward other cochannel mobiles. A performance analysis of a null-steering algorithm is presented in [39].

D. Optimal Beam Forming

The null-steering scheme described in the previous section requires knowledge of the directions of interference sources, and the beam former using the weights estimated by this scheme does not maximize the output SNR. The optimal beam-forming method described in this section overcomes these limitations.

Let an L-dimensional complex vector $\hat{\underline{w}}$ represent the weights of the beam former shown in Fig. 2, which maximizes the output SNR. For an array that is not constrained, an expression for $\hat{\underline{w}}$ is given by [17], [24], [40], [41]

$$\hat{\underline{w}} = \mu_0 R_N^{-1} \underline{s}_0 \qquad (39)$$

where R_N is the array correlation matrix of the noise alone, that is, it does not contain any signal arriving from the look direction (ϕ_0, θ_0), and μ_0 is a constant. For an array constrained to have a unit response in the look direction, this constant becomes

$$\mu_0 = \frac{1}{\underline{s}_0^H R_N^{-1} \underline{s}_0} \qquad (40)$$

leading to the following expression for the weight vector:

$$\hat{\underline{w}} = \frac{R_N^{-1} \underline{s}_0}{\underline{s}_0^H R_N^{-1} \underline{s}_0}. \qquad (41)$$

As the weights are computed using NAME, the processor with these weights is referred to as the NAME processor [42]. It is also known as the ML filter [43], as it finds the ML estimate of the power of the signal source, assuming all sources as interferences. It should be noted R_N may not be invertible when the background noise is very small. In that case, it becomes a rank deficient matrix.

In practice, when the estimate of the noise-alone matrix is not available, the total R (signal plus noise) is used to estimate the weights and the processor is referred to as the SPNMI processor. An expression for the weights for this case is given by

$$\hat{\underline{w}} = \frac{R^{-1} \underline{s}_0}{\underline{s}_0^H R^{-1} s_0}. \qquad (42)$$

These weights are the solution of the following optimization problem:

$$\begin{aligned} \underset{\underline{w}}{\text{minimize}} & \quad \underline{w}^H R \underline{w} \\ \text{subject to} & \quad \underline{w}^H \underline{s}_0 = 1. \end{aligned} \qquad (43)$$

Thus, the processor weights are selected by minimizing the mean output power of the processor while maintaining unity response in the look direction. The constraint ensures that the signal passes through the processor undistorted. Therefore, the output signal power is the same as the look-direction source power. The minimization process then minimizes the total noise, including interferences and uncorrelated noise. Minimizing the total output noise while keeping the output signal constant is the same as maximizing the output SNR.

It should be noted that the weights of the NAMI processor and the SPNAMI processor are identical, and in the absence of errors, the processor performs identically in both cases. This fact can be proved as follows.

The Matrix Inversion Lemma for an invertible matrix A and a vector $\underline{x}$ states that

$$(A + \underline{x}\,\underline{x}^H)^{-1} = A^{-1} - \frac{A^{-1}\underline{x}\,\underline{x}^H A^{-1}}{1 + \underline{x}^H A^{-1}\underline{x}}. \qquad (44)$$

Since

$$R = p_s \underline{s}_0 \underline{s}_0^H + R_N \qquad (45)$$

it follows from the Matrix Inversion Lemma that

$$R^{-1} = R_N^{-1} - \frac{p_s R_N^{-1} \underline{s}_0 \underline{s}_0^H R_N^{-1}}{1 + \underline{s}_0^H R_N^{-1} \underline{s}_0 p_s}. \qquad (46)$$

A substitution for R^{-1} in (42) and algebraic manipulation leads to the expression for weights given by (41), showing that the two expressions are identical.

The processor with these weights is referred to as the optimal processor. The output SNR $\hat{\alpha}$ of the optimal processor is given by [29]

$$\hat{\alpha} = p_s \underline{s}_0^H R_N^{-1} \underline{s}_0. \tag{47}$$

For a special case of the noise environment when no direction interference is present, a simple calculation yields

$$\hat{\underline{w}} = \frac{\underline{s}_0}{L}. \tag{48}$$

Thus, the weights of the optimal processor in the absence of errors are the same as those of the conventional processor, implying that the conventional processor is the optimal processor for this case. The output SNR and the array gain $\hat{G}$ of the optimal processor for this case are, respectively, given by

$$\hat{\alpha} = \frac{p_s L}{\sigma_n^2} \tag{49}$$

and

$$\hat{G} = L. \tag{50}$$

For the case of one-directional interference of power p_I, the expression for the output SNR becomes

$$\hat{\alpha} = \frac{p_s L}{\sigma_n^2} \frac{\rho + \frac{\sigma_n^2}{p_I L}}{1 + \frac{\sigma_n^2}{p_I L}} \tag{51}$$

and the array gain is given by

$$\hat{G} = \frac{p_I L}{\sigma_n^2} \frac{\left(1 + \frac{\sigma_n^2}{p_I}\right)\left(\rho + \frac{\sigma_n^2}{p_I L}\right)}{1 + p_I} \tag{52}$$

where

$$\rho = 1 - \frac{\underline{s}_0^H \underline{s}_I \underline{s}_I^H \underline{s}_0}{L^2} \tag{53}$$

is a scalar quantity and depends upon the direction of the interference relative to the signal source and the array geometry [29]. It follows from (23) and (53) after rearrangement that

$$\rho = 1 - \underline{w}_c^H \underline{s}_I \underline{s}_I^H \underline{w}_c. \tag{54}$$

Thus, this parameter is characterized by the weights of the conventional processor. As this parameter characterizes the performance of the optimal processor, it implies that the performance of the optimal processor in terms of its interference cancellation capability depends to a certain extent upon the response of the conventional processor to the interference. This fact has been further highlighted in [44] and [45].

An interesting special case is one where the interference is much stronger compared to the background noise, $p_I \gg \sigma_n^2$. For this case, these expressions may be approximated as

$$\hat{\alpha} \cong \frac{p_s L \rho}{\sigma_n^2} \tag{55}$$

and

$$\hat{G} \cong \frac{p_I L \rho}{\sigma_n^2}. \tag{56}$$

When interference is away from the main lobe of the conventional processor $\rho \approx 1$, it follows that the output SNR of the optimal processor in the presence of a strong interference is the same as that of the conventional processor in the absence of interference, implying that the processor has almost completely canceled the interference, yielding a very large array gain.

The performance of the processor in terms of its output SNR and the array gain is not affected by the look-direction constraint, as it only scales the weights. Therefore, the treatment presented above is valid for the unconstrained processor.

For the optimal beam former to operate as described above and to maximize the SNR by canceling interferences, the number of interferences must be less than or equal to $L - 2$, as an array with L elements has $L - 1$ degrees of freedom and one has been utilized by the constraint in the look direction. This may not be true in a mobile communications environment due to existence of multipath arrivals, and the array beam former may not be able to achieve the maximization of the output SNR by suppressing every interference. As argued in [46], however, the beam former does not have to suppress interferences to a great extent and cause a vast increase in the output SNR to improve the performance of a mobile radio system. An increase of a few decibels in the output SNR can make a large increase in the channel capacity of the system possible.

In mobile communications literature, the optimal beam former is often referred to as the optimal combiner. Discussion on the use of the optimal combiner to cancel interferences and to improve the performance of mobile communications systems can be found in [46]–[49].

It should be noted that the optimal beam former described in this section, also known as the MVDR beam former, does not require the knowledge of the directions and power levels of the interferences as well as the level of the background noise power to maximize the output SNR. It requires only the direction of the desired signal. In the next section, a processor is described that requires a reference signal instead of the desired signal direction.

E. Optimization Using Reference Signal

A narrow-band beam-forming structure that employs a reference signal [24], [27], [28], [50]–[52] to estimate the weights of the beam former is shown in Fig. 4. The array output is subtracted from an available reference signal $r(t)$ to generate an error signal $\varepsilon(t) = r(t) - \underline{w}^H \underline{x}(t)$, which is
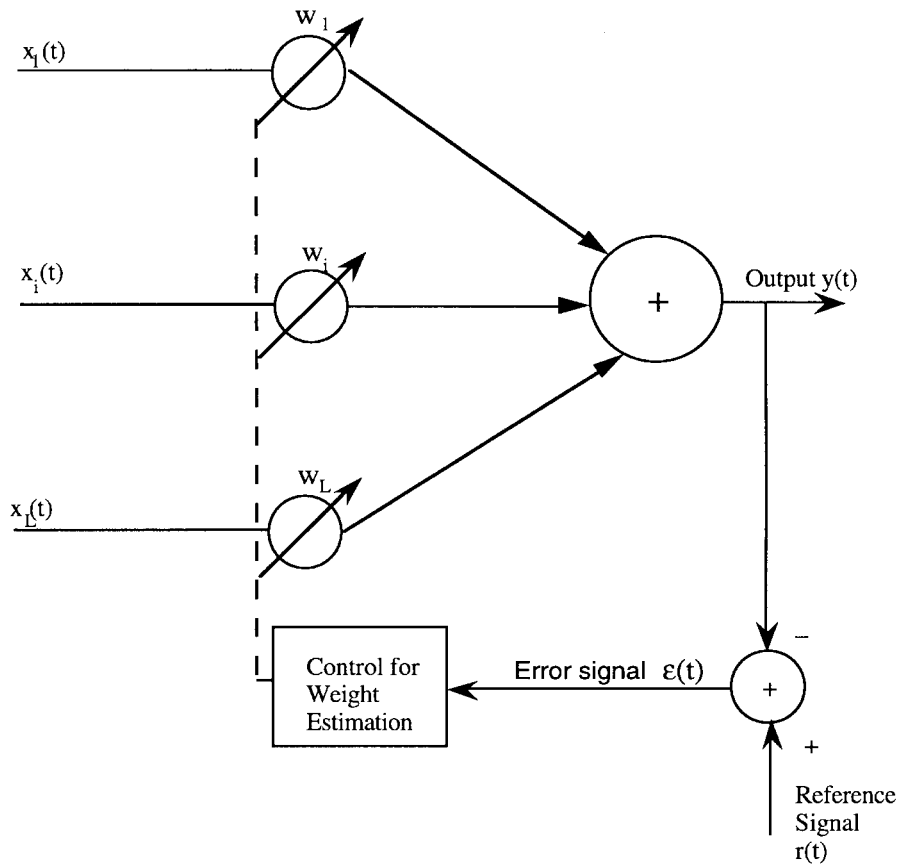
Fig. 4. Structure of narrow-band beam former using a reference signal.

used to control the weights. Weights are adjusted such that the MSE between the array output and the reference signal is minimized. The MSE is given by

$$
\begin{aligned}
\text{MSE} &= E[|\varepsilon(t)|^2] \\
&= E[|r(t)|^2] + \underline{w}^H R \underline{w} - 2 \underline{w}^H \underline{z}
\end{aligned}
\tag{57}
$$

where

$$
\underline{z} = E[\underline{x}(t) \quad r(t)]
\tag{58}
$$

is the correlation between the reference signal and the array signals vector $\underline{x}(t)$.

The MSE surface is a quadratic function of $\underline{w}$ and is minimized by setting its gradient with respect to $\underline{w}$ equal to zero, yielding the well-known Wiener–Hoff equation for the optimal weight vector

$$
\underline{\hat{w}}_{\text{MSE}} = R^{-1} \underline{z}.
\tag{59}
$$

The MMSE of the processor, also known as the Wiener filter, using these weights is given by

$$
\text{MMSE} = E[|r(t)|^2] - \underline{z}^H R^{-1} \underline{z}.
\tag{60}
$$

The scheme may be employed to acquire a weak signal in the presence of a strong jammer, as discussed in [50], by setting the reference signal to zero and initializing the weights to provide an omnidirectional pattern. The process starts to cancel strong interferences first and the weak signal later. Thus, intuitively, there is expected to be a time when the output would consist of the signal that has not been canceled but strong interferences have been reduced.

When an adaptive scheme (discussed in Section III-B) is used to estimate $\underline{\hat{w}}_{\text{MSE}}$, the strong jammer gets canceled first as the weights are adjusted to put a null in that direction to leave signal-to-jammer ratio sufficient for acquisition.

Arrays using reference signals equal to zero to adjust weights are referred to as power-inversion adaptive arrays [53]. The MSE minimization scheme (the Wiener filter) is a closed-loop method compared to the open-loop scheme of MVDR (the ML filter) described in the previous section. In general, the Wiener filter provides higher output SNR compared to the ML filter in the presence of a weak signal source. As the input signal power becomes large compared to the background noise, the two processors give almost the same results [54]. This result is supported by a simulation study for a two-vehicle mobile communications situation in [55]. The increased SNR by the Wiener filter is achieved at the cost of signal distortion caused by the filter. It should be noted that the optimal beam former does not distort the signal.

The required reference signal for the Wiener filter may be generated in a number of ways, depending upon the application. In digital mobile communications, a synchronization signal may be used for initial weight estimation, followed by the use of detected signal as a reference signal. In systems using a TDMA scheme, a sequence that is user specific may be a part of every frame for this purpose [56].

The use of a known symbol in every frame has also been suggested [57]. In other situations, the use of an antenna for this purpose has been examined to show the suitability to provide a reference signal [57].

Studies of mobile communications systems using reference signals to estimate array weights have also been reported in [58]–[60].

F. Beam-Space Processing

In contrast to element-space processing, where signals derived from each element are weighted and summed to produce the array output, beam-space processing is a two-stage scheme where the first stage takes the array signals as input and produces a set of multiple outputs, which are then weighted and combined to produce the array output. These multiple outputs may be thought of as the output of multiple beams. The processing done at the first stage is by fixed weighting of the array signals and amounts to produce multiple beams steered in different directions. These weights are normally not adaptive, that is, they are not adjusted during adaption cycle. The weights applied to different beam outputs to produce the array outputs are optimized to meet a specific optimization criteria and are adjusted during the adaption cycle.

In general, for an L-element array, a beam-space processor consists of a main beam steered in the signal direction and a set of not more than $L - 1$ secondary beams. The weighted output of the secondary beams is subtracted from the main beam. The weights are adjusted to produce an estimate of the interference present in the main beam. The subtraction process then removes this interference. The secondary beams, also known as auxiliary beams, are designed such that they do not contain the desired signal from the look direction to avoid the signal cancellation in the subtraction process. A general structure of such a processor is shown in Fig. 5. Beam-space processors have been studied under many different names, including Howells–Applebaum array [24], [51], [61], GSC [62], [63], partitioned processor [64], [65], partially adaptive arrays [66]–[72], PIC [73]–[77], adaptive-adaptive arrays [78], and multiple-beam antennas [79]–[81].

The pattern of the main beam is normally referred to as the quiescent pattern, and is chosen such that it has a desired shape. For a linear array of equispaced elements with equal weighting, the quiescent pattern has the shape of $\sin Lx/\sin x$, with L being the number of elements in the array, whereas for Chebyshev weighting (the weighting dependent upon the coefficients of the Chebyshev polynomial), the pattern has equal side-lobe levels [82]. The pattern of the main beam may be adjusted by various forms of constraints [51] and pattern synthesis techniques, which are discussed in [83]–[87] and the references therein.

There are many schemes to generate the outputs of auxiliary beams such that no signal from the look direction is contained in them, that is, the beams have nulls in the look direction. In its simplest form, this can be accomplished by subtracting the array signals from presteered adjacent pairs [26], [88]. This relies on the fact that the component of the
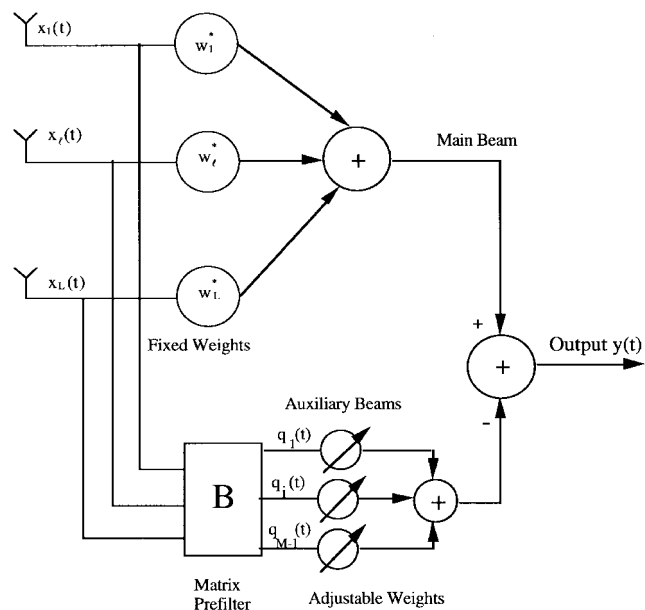

Fig. 5. Structure of a general beam-space processor.

array signals induced from a source in the look direction is identical after the presteering, and this gets canceled in the subtraction process from the adjacent pairs. The process can be generalized to produce $M - 1$ beams from L-element array signals $x(t)$ using a matrix B such that

$$\underline{q} = \underline{x}^H(t)B \tag{61}$$

where $M - 1$ dimensional vector $\underline{q}(t)$ denotes the outputs of $M - 1$ beams and the matrix B, referred to as the blocking matrix or the matrix prefilter, has the property that its $M - 1$ columns are linearly independent and that the sum of the elements of each column equals zero, implying that $M - 1$ beams are independent and have nulls in the look direction. For an array that is not presteered, the matrix needs to satisfy

$$\underline{s}_0^H B = 0 \tag{62}$$

where $\underline{s}_0$ is the steering vector associated with the look direction.

It is assumed in the above discussion that $M \leq L$, implying that the number of beams is less than or equal to the number of elements in the array. When the number of beams is equal to the number of elements in the array, the processing in the beam space has not reduced the degree of freedom of the array, that is, its null-forming capability has not been reduced. In this sense, these arrays are fully adaptive and have the same capabilities as those of the array using element-space processing. In fact, in the absence of errors, both processing schemes produce identical results. On the other hand, when the number of beams is less than the number of elements, the arrays are referred to as partially adaptive. The null-steering capabilities of these arrays have reduced to that equal to the number of auxiliary beams. When adaptive schemes are used to estimate the weights, the convergence is generally faster for these arrays.

The MSE for these arrays, however, is also high compared to that of the fully adaptive arrays [89].

These arrays are useful in situations where the number of interferences is much less than the number of elements. They offer a computational advantage over element-space processing, as one needs only to adjust $M - 1$ weights compared to L weights for the element-space case with $M < L$. Moreover, beam-space processing requires less computation than the element-space case to calculate the weights in general, as it solves an unconstrained optimization compared to the constrained optimization problem solved in the later case. It should be noted that for the element-space processing case, the constraints on the weights are imposed to prevent the signal arriving from the look direction from being distorted and to make the array more robust against errors. For the beam-space case, these are transferred to the main beam, leaving the adjustable weights free from constraints.

A performance comparison of an element-space processor and a beam-space processor for the case of a single interference case is presented in [90]. The beam-space processor considered is a single auxiliary beam processor, referred to as the PIC processor, which is useful for canceling single interference only. The study shows that in the absence of errors, both processors produce identical results, whereas in the presence of look-direction errors, the beam-space processor produces superior performance. The situation arises when the known direction of the signal is different from the actual direction.

The weights of the processor are constrained with the knowledge of the look direction. When the actual signal direction is different from the one that is used to constrain weights, the element-space processor cancels this signal as if it was an interference close to the look direction. The beam-space processor, on the other hand, is designed to have the main beam steered in the known look direction, and the auxiliary beams are formed to have null in this direction. The response of the main beam does not alter much away from the look direction, and thus the signal level in the main beam is not affected. Similarly, when a null of the auxiliary beams is placed in the known look direction, a very small amount of the signal leaks in the auxiliary beam due to a source very close to the null. Thus, the subtraction process does not affect the signal level in the main beam, yielding a very small signal cancellation in beam-space processing compared to element-space processing. For details of the effect of other errors on beam-space processors, particularly GSC, see, for example, [91].

The auxiliary beam-forming techniques other than the use of a blocking matrix (described above) includes formation of $M - 1$ orthogonal beams and formation of beams in the direction of interferences if known. The beams are referred to as orthogonal beams to imply that the weight vectors used to form beams are orthogonal, that is, their dot product is equal to zero. The eigenvectors of R taken as weights to generate auxiliary beams fall into this category. In situations where the DOA's of interferences are known, the formation of beams pointed in these directions may lead to more efficient interference cancellation [78], [92].

The auxiliary beam outputs are weighted and summed, and the result is subtracted from the main beam output to cancel the unwanted interference present in the main beam. The weights are adjusted to cancel the maximum possible interference. This is normally done by minimizing the total mean output power after subtraction by solving the unconstrained optimization problem, and leads to maximization of the output SNR in the absence of the desired signal in auxiliary channels. The presence of the signal in these channels causes signal cancellation from the main beam, along with interference cancellation. A detailed discussion on the principles of signal cancellation in general and some possible cures is given in [28], [52], and [93].

Use of multiple-beam array-processing techniques for mobile communications has been reported in various studies [94]–[98], including development of a 16-element array system using digital hardware to study its feasibility [99].

G. Broad-Band Beam Forming

The beam-former structure of Fig. 2 discussed earlier is for narrow-band signals. As the signal bandwidth increases, the performance of the beam former using this structure starts to deteriorate [100]. For processing broad-band signals, a TDL structure, shown in Fig. 6, is normally used [100]–[108]. A lattice structure consisting of a cascade of J simple lattice filters sometimes is also used [109]–[113], offering some processing advantages.

The steering delays in front of each element in Fig. 6 are pure time delays and are used to steer the array in a given look direction (ϕ_0, θ_0). If $\tau_\ell(\phi_0, \theta_0)$ denotes the time taken by the plane wave arriving from direction (ϕ_0, θ_0) and measured from the reference point to the ℓth element, then the steering delay $T_\ell(\phi_0, \theta_0)$ may be selected using

$$T_\ell(\phi_0, \theta_0) = T_0 + \tau_\ell(\phi_0, \theta_0), \quad \ell = 1, 2, \cdots, L \quad (63)$$

where T_0 is a bulk delay such that $T_\ell(\phi_0, \theta_0) > 0, \forall_\ell$.

If $s(t)$ denote the signal induced, on an element present at the center of the coordinate system, due to a broad-band source of power density $S(f)$ then the output of the ℓth sensor pre-steered in (ϕ_0, θ_0), is given by

$$x_\ell(t) = s(t + \tau_\ell(\phi, \theta) - T_\ell(\phi_0, \theta_0)). \quad (64)$$

For a source in (ϕ_0, θ_0), it becomes

$$x_\ell(t) = s(t - T_0) \quad \ell = 1, 2, \cdots, L \quad (65)$$

yielding identical waveforms after pre-steering delays.

The TDL structure shown in the figure following the steering delay on each channel is a FIR filter. The coefficients of these filters are constrained to specify the frequency response in the look direction. It should be noted that these coefficients are real compared to the complex weights of the narrow-band processor.

Let $\underline{w}$, defined by

$$\underline{w} = [\underline{w}_1, \underline{w}_2, \cdots, \underline{w}_J]^T \quad (66)$$
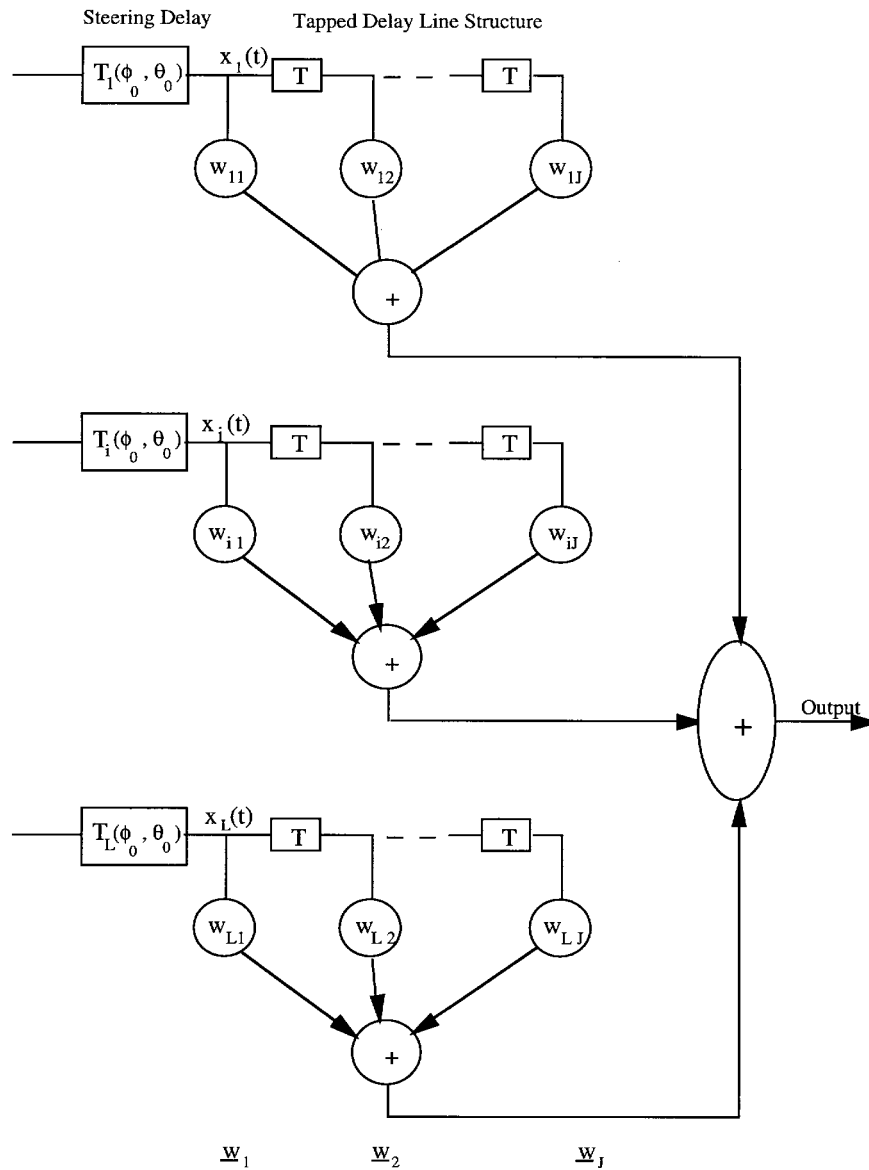
Fig. 6. Broad-band beam-former structure using TDL filter.

denote LJ coefficients of the filter structure with $\underline{w}_m$ denoting the L coefficients after the $(M-1)$th tap. The mean out power of the beam former for a given $\underline{w}$ is given by

$$P(\underline{w}) = \underline{w}^T R \underline{w} \tag{67}$$

where the $LJ \times LJ$-dimensional real matrix R denotes the array correlation matrix, with its elements representing the correlation between various tap outputs. The correlation between the outputs of the $(\ell-1)$th tap on the mth channel and the $(k-1)$th tap on the nth channel is given by

$$(R_{m,n})_{\ell,k} = \rho[(m-n)T + T_\ell(\phi_0,\theta_0) - T_k(\phi_0,\theta_0) \\ + \tau_k(\phi,\theta) - \tau_\ell(\phi,\theta)] \tag{68}$$

with $\rho(\tau)$ denoting the correlation function

$$\rho(\tau) = E[s(t)s(t+\tau)]. \tag{69}$$

It is related to the spectrum of the signal by the Fourier transform, that is

$$\rho(\tau) = \int_{-\infty}^{\infty} S(f)e^{j2\pi f\tau}\, df. \tag{70}$$

Thus, from the knowledge of the spectra of sources and their DOA's, the correlation matrix may be calculated. In practice, this can also be estimated by measuring signals at the output of various taps.

In situations where one is interested in finding coefficients such that the beam former cancels the directional interferences and has the specified response in the look direction, the following beam-forming problem is normally considered:

$$\underset{\underline{w}}{\text{minimize}} \quad \underline{w}^T R \underline{w} \tag{71}$$

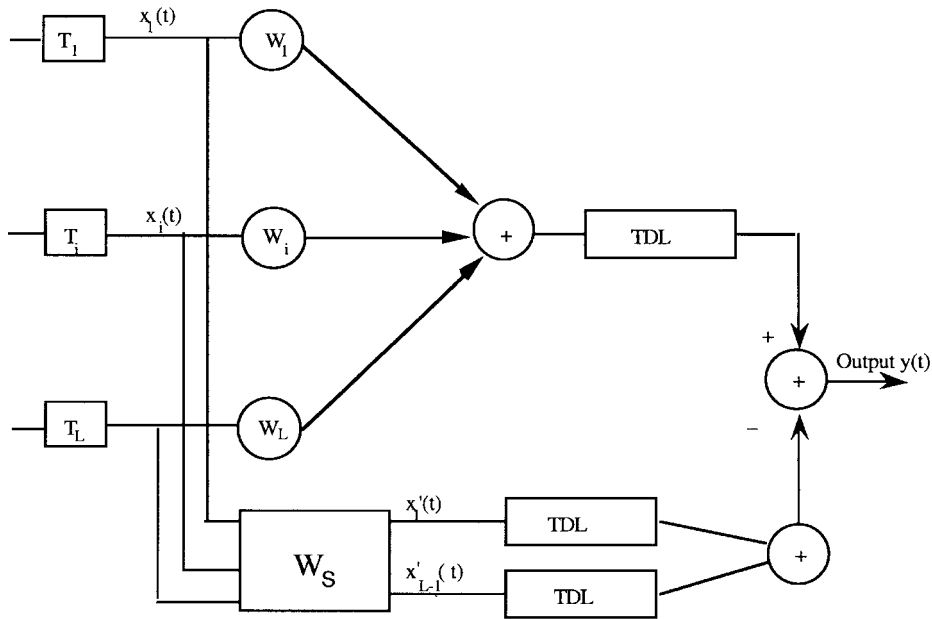$$\text{subject to} \quad C^T \underline{w} = \underline{F} \tag{72}$$

Fig. 7. Structure of partitioned realization of the broad-band beam former.

where $\underline{F}$ is a J-dimensional vector that specifies the frequency response in the look direction and C is an $LJ \times J$ constraint matrix. For a point constraint in the look direction

$$C = \begin{bmatrix} \underline{1} & & & 0 \\ & \underline{1} & & \\ & & \cdot & \\ & & & \cdot \\ & & & \cdot \\ 0 & & & \underline{1} \end{bmatrix} \tag{73}$$

with $\underline{1}$ denoting the L-dimensional vector of 1s. Let $\hat{\underline{w}}$ denote the solution of the above problem. It is given by [25]

$$\hat{\underline{w}} = R^{-1}C(C^T R^{-1} C)^{-1} \underline{F}. \tag{74}$$

The point-constraint minimization problem specifies J constraints on the weights such that the sum of L weights on all the channels before the jth delay is equal to F_j. For all pass frequency responses in the look direction, all but one F_j, $j = 1, \cdots, J$ are selected to be equal to zero. For j close to $(J+1)/2$, F_j is taken to be unity. Thus, the constraints specify that the sum of the weights across the array is zero except for one near the middle of the filter, which is equal to unity.

The implication of these constraints is that the array pattern has a unity response in the look direction. This pattern can be broadened by specifying additional constraints, such as derivative constraints [114]–[116], along with the constraints discussed above. The derivative constraints set the derivatives of the power pattern with respect to θ and ϕ equal to zero in the look direction. The higher the order of derivatives, that is, the first order, second order, etc., the broader the beam in the look direction normally becomes. A broader beam is useful when the actual signal direction and the known direction of the signal is not precisely the same. In such situations the processor with the

point constraint in the known direction of the signal would cancel the desired signal as if it were an interference. The other directional constraints to improve the performance of the beam former in the presence of the look-directional constraints include multiple linear constraints [117], [118] and inequality constraints [119]–[121].

A set of nondirectional constraints to improve the performance of the beam former under look-direction errors is discussed in [122]. These are referred to as correlation constraints, which use the known characteristics of the desired signal to estimate an LJ-dimensional correlation vector $\underline{r}_d$ between the desired signal and the array signal vector. The beam-forming problem using these constraints becomes

$$\text{minimize} \quad \underline{w}^T R w \tag{75}$$

$$\text{subject to} \quad \underline{r}_d^T \underline{w} = \rho_0 \tag{76}$$

where ρ_0 is a scalar constant that specifies the correlation between the desired signal and the array output.

Application of broad-band beam-forming structures using TDL filters to mobile communications has been considered in [56] and [123]–[125] to overcome multipath fading and large delay spread in a TDMA as well as a CDMA system.

H. Partitioned Realization

The broad-band beam-former structure shown in Fig. 6 is sometimes referred to as an element-space processor or direct form of realization, compared to a beam-space processor or partitioned form of realization, as shown in Fig. 7. The structure shown in Fig. 7 is discussed below for a point constraint, that is, the response is constrained to be unity in the look direction. A discussion of partitioned realization for derivative constraints may be found in [126].

The steering delays are used to align the waveform arriving from the look direction, as discussed. The array

signals after the steering delays are passed through two sections. The top section consists of a broad-band conventional beam with required frequency response obtained by selecting the coefficients of the FIR filter. Signals from all of the channels are equally weighted and summed. For this realization to be equivalent to the direct form of realization, all the weights need to be equal to $1/L$, and the filter coefficients $F_j, \ j = 1, 2, \cdots, J$ need to be specified as before. Furthermore, the output of the upper section is given by

$$y_c(t) = \sum_{k=0}^{J-1} F_{k+1} y(t - Tk) \qquad (77)$$

with

$$y(t) = \frac{\underline{x}^T(t) \underline{1}}{L}. \qquad (78)$$

The matrix prefilter shown in the lower section is designed to block the signal arriving from the look direction. Since these signal waveforms after the steering delays are alike, it can be achieved by selecting the matrix W_s such that the sum of each of its rows is equal to zero. For the partitioned processor to have the same degree of freedom as that of the direct form, the $L-1$ rows of the matrix W_s need to be linearly independent. The output $\underline{x}'(t)$ after the matrix prefilter is an $L-1$-dimensional vector given by

$$\underline{x}'(t) = W_s \underline{x}(t) \qquad (79)$$

and can be thought of as the outputs of $L-1$ beams, which are then shaped by the coefficients of the FIR filter of each TDL section. Let an $L-1$-dimensional vector $\underline{a}_k$ denote these coefficients before the kth delay. The output of the lower filter is then given by

$$y_a(t) = \sum_{k=0}^{J-1} \underline{a}_k^T \underline{x}'(t - kT). \qquad (80)$$

These coefficients are selected by minimizing the mean output of the processor, that is

$$\underset{\underline{a}_k}{\text{minimize}} \quad E\big[(y_c(t) - y_a(t)^2)\big]. \qquad (81)$$

The performance of the broad-band arrays as a function of the number of various parameters, such as the number of taps, tap spacing, array geometry, array aperture, and signal bandwidth, has been considered in the literature [101]–[108] to understand their influence on the behavior of the arrays. An analysis [101] of broad-band arrays using eigenvalues of R indicates that the product of the array aperture and the FBW of the signal is an important parameter of the broad-band array in determining its performance. The FBW is defined as the ratio of the bandwidth to the center frequency of the signal. It is shown that the number of taps required on each element depends upon this parameter as well as on the shape of the array, with more taps needed for a complex shape. A study [102], [103] of the SNR as a function of intertap spacing indicates that there is a range of intertap spacing that yields close to

the maximum attainable SNR and depends upon the FBW of the signal. This range includes a quarter-wavelength spacing at the center frequency f_0. The quarter-wavelength spacing produces a $90°$ phase shift at f_0 and is equal to $1/4f_0$. By measuring the tap spacing as a multiple of this delay, it is indicated that the intertap spacing with multiple around $1/$FBW yields close to the highest attainable SNR. With the multiple between $1/$FBW to $4/$FBW, one needs a larger number of taps for an equivalent performance.

A study of the jamming rejection capability [104] and the tracking performance of the array in a nonstationary environment [105] also indicates that when tap spacing is measured in terms of the center frequency of the signal, the best performance is achieved when the spacing is $1/4f_0$. For this tap spacing, R has less eigenvalue spread, which is the reason for this performance. The eigenvalue spread of a matrix indicates the range of values its eigenvalues take. A larger ratio of the largest eigenvalue to the smallest eigenvalue indicates a larger spread.

The TDL filter tends to increase the degrees of freedom of the array, which may be traded against the number of elements such that an array with L elements is able to suppress more than $L-1$ directional interferences, provided their center frequencies are not the same and fall within the FBW of the signal [107].

Though the TDL structure with constrained optimization is the commonly used structure for broad-band array signal processing, alternative methods have been proposed. These include:

1) adaptive nonlinear schemes, which maximize SNR subject to additional constraints [127];

2) a variation of a Davis beam former [88], which adapts one filter at a time to speed up convergence [128];

3) a composite system, which also utilizes a derivative of beam pattern in the feedback loop to control the weights [129] to reject wideband interference;

4) optimum filters, which specify rejection response [87];

5) a master and slave processor with broad-beam capabilities without derivative constraints [130];

6) a hybrid method that uses an orthogonal transformation on data available from the TDL structure before applying weights [131] to improve its performance in multipath environment;

7) weighted Chebyshev method [134];

8) two-sided correlation transformation method [135].

I. Frequency-Domain Beam Forming

A general structure of the element-space frequency-domain processor is shown in Fig. 8, where broad-band signals from each element are transformed into frequency domain using the FFT and each frequency bin is processed by a narrow-band processor structure. The weighted signals from all elements are summed to produce an output at each bin. The weights are selected by independently minimizing
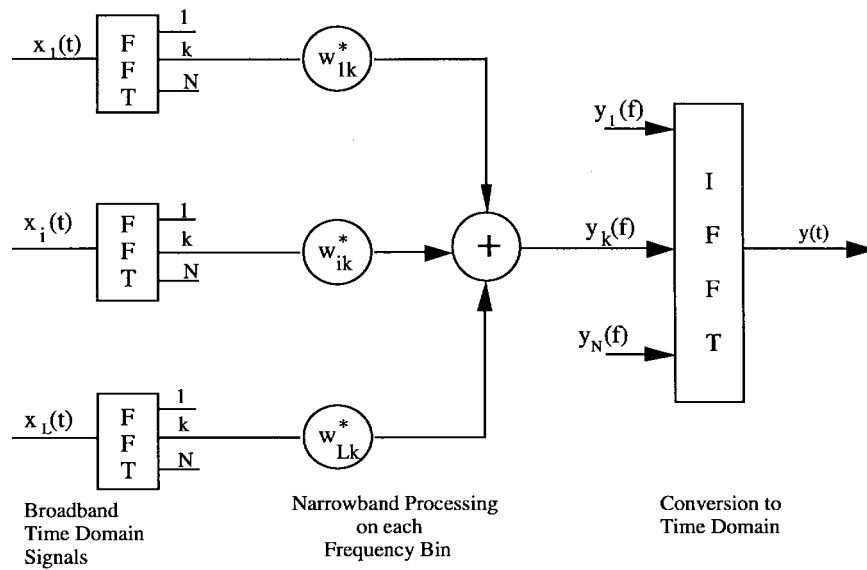
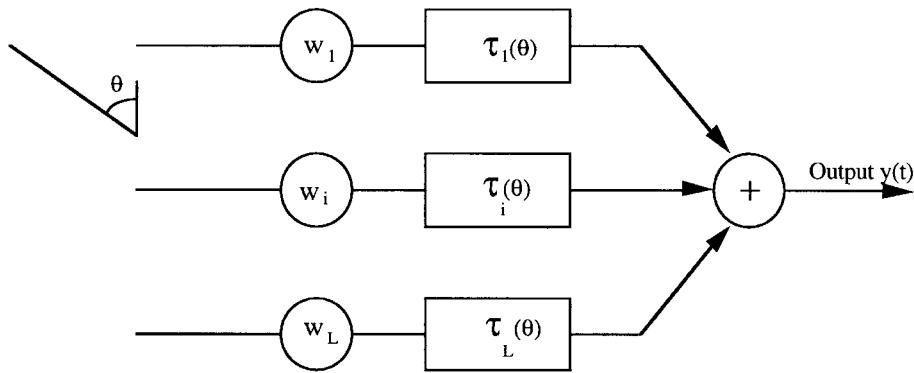**Fig. 8.** Element-space frequency-domain processor structure.

Fig. 9. Delay-and-sum beam former.

the mean output power at each frequency bin subject to steering-direction constraints. Thus, the weights required for each frequency bin are selected independently, and this selection may be performed in parallel, leading to a faster weight update. When adaptive algorithms such as the LMS algorithm (discussed in Section III-B) is used for weight update, a different step size may be used for each bin, leading to faster convergence.

Various aspects of frequency-domain beam forming are reported in the literature [136]–[150]. The performance of the time- and frequency-domain processors are the same only when the signals in different frequency bins are independent. This independence assumption is mostly made in the study of frequency-domain beam forming. When this assumption does not hold, the frequency-domain beam former may be suboptimal. Some of the tradeoffs and comparisons of the two processors may be found in [136] and [149].

A study of the frequency-domain algorithm [140] for coherent signals indicates that the frequency-domain method is insensitive to the sampling rate and may be able to reduce the effects of element malfunctioning on the beam pattern. A study in [141] shows that due to its modular parallel

structure, beam forming in the frequency domain is well suited for VLSI implementation and is less sensitive to the coefficient quantization. The computational advantage of the frequency-domain method for bearing estimation is discussed in [144], [146], and [150], and the advantage for correlated data is considered in [145] and [148]. A general treatment of time- and frequency-domain realization with a view to comparing the structure of various algorithms of weight estimation in a unified manner is provided in [139].

J. Digital Beam Forming

Consider the analog beam-former structure shown in Fig. 9, where the signals from each element are weighted, delayed, and summed to form the beam output

$$y(t) = \sum_{i=1}^{L} w_i x_i(t - \tau_i(\theta)). \tag{82}$$

The delays are adjusted such that the signals induced from a given direction, where the beam needs to be pointed, are aligned after the delays. This aspect of beam steering was discussed in detail earlier. The weights are adjusted to shape the beam.
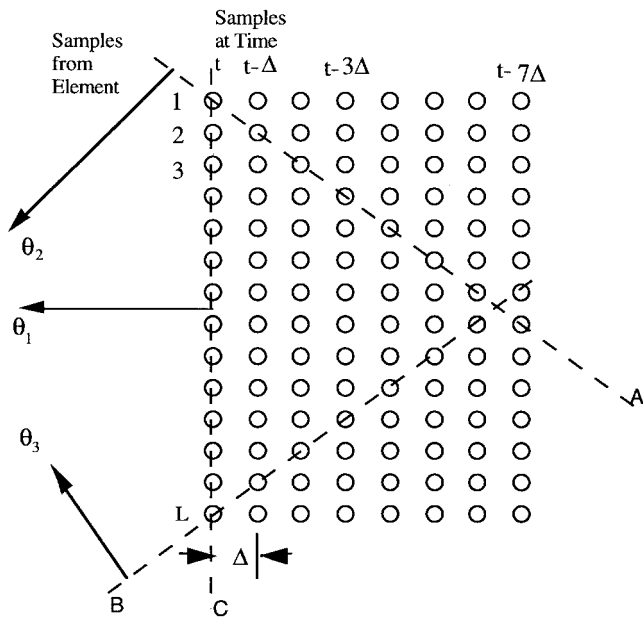
Fig. 10. Digital beam-forming process.

Fig. 11. Effect of sampling on digital beam forming.

In digital beam forming [151]–[164], the weighted signals from each element are sampled and stored, and beams are formed by summing the appropriate samples such that the required delay is incorporated by this process. It requires each delay as an integer multiple of the sampling interval Δ. The process is shown in Fig. 10 for a linear array of equispaced elements, where it is desired that a beam is formed in direction θ_2. Let the direction be such that

$$\tau_i(\theta_2) = (i-1)\Delta. \tag{83}$$

Thus, the signal from the ith element needs to be delayed by $(i-1)\Delta$ seconds. This may be accomplished by selecting the samples for summing (as shown in Fig. 10 by the line marked with symbol A). Similarly, a beam may be steered in direction θ_3 by summing the samples connected by the line marked with symbol B in Fig. 10, where the signals from the ith element need to be delayed by $(L-i)\Delta$ seconds. The beam formed in direction θ_1, by summing the samples connected by the line marked with symbol C, does not require any delay.

It follows from the above discussion that using this process, one can only form beams in those directions that require delays equal to some integer multiple of the sampling interval, that is

$$\tau_i(\theta) = k_i\Delta \tag{84}$$

where k_i, $i = 1, 2, \cdots, L$ are integers.

The number of discrete directions where a beam can be pointed exactly increases with increased sampling, as shown in Fig. 11, where the sampling interval is $\Delta/2$. The figure shows that additional beams in directions θ_4 and θ_5 may be formed. These exact beams are normally referred to as synchronous or natural beams [152], and it is possible to form a number of these beams simultaneously using a separate summing network for each beam.
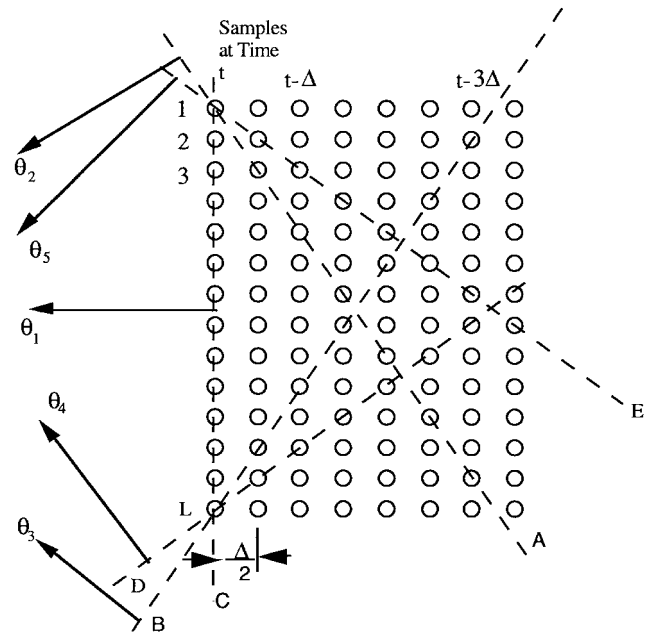
The practical requirement of an adequate set of directions where simultaneous beams need to be pointed implies that the array signals be sampled at much higher rates than required by Nyquist criterion to reconstruct the waveform back from the samples [165]. The high sampling rate means a large number of storage requirements along with high-speed input-output devices, analog-to-digital converters, and large bandwidth cables [152].

The requirement of high sampling rates may be overcome by digital interpolation [152], [157], [163]. This process basically simulates the samples generated by high sampling rates and thus increases the effective sampling rate. It works by sampling the array signal at the Nyquist rate or higher and by padding between each sample with zeros to form a new sequence. The number of zeros padded decides the effective sampling rate. For a sampling rate to increase L-fold, $L - 1$ zeros are padded to create a sequence as large as if it were created by sampling at high speed. The padded sequences then are used for digital beam forming by selecting appropriate samples as required, and the beam output is passed through an FIR filter to remove unwanted spectra. This filter is normally referred to as an interpolation filter. The beams formed by interpolation beam formers have slightly higher side-lobe levels.

A tutorial introduction to digital-interpolation beam formers is given in [152], whereas some additional fundamentals of digital-array processing may be found in [155]. A comparison of many approaches to digital beam-forming implementations is discussed in [156] and [159], showing how a real-time implementation is a tradeoff between various conflicting requirements of hardware complexities, memory, and system performance.

The shape of a beam, particularly its beamwidth, is controlled by the size of the array. Generally, a narrow beam results from a larger array. In practice, the array size is fixed

and its extent is limited. A process known as extrapolation may be used [158] during digital beam forming to simulate a large array extent resulting in improved beam pattern. As the interpolation increases the effective sampling rate, the extrapolation extends the effective array length. More information on signal extrapolation schemes may be found in [165]–[170].

Digital beam-forming techniques for mobile satellite communications are examined in [95] by studying a configuration of a digital beam-forming system capable of working in transmit and receive mode. Digital beam forming for mobile satellite communications has also been reported in [59], [95], [171], and [172]. An introduction to digital beam forming for mobile communications may be found in [173].

K. Eigenstructure Method

As discussed previously, the eigenvalues of R can be divided into two sets when the environment consists of uncorrelated directional sources and uncorrelated white noise.

The largest M eigenvalues correspond to M directional sources, and the eigenvectors associated with these eigenvalues are normally referred to as signal eigenvectors. The $L–M$ smallest eigenvalues are equal to the background noise power, and the eigenvectors associated with these eigenvalues are known as noise eigenvectors.

The eigenvectors of R are orthogonal to each other and thus may be thought of as spanning an L-dimensional space. This space may be divided into two orthogonal subspaces. The subspace spanned by signal eigenvectors is referred to as the signal subspace, whereas the subspace spanned by the noise eigenvectors is referred to as the noise subspace.

The signal subspace is also spanned by M steering vectors associated with M directional sources. This fact is exploited by eigenstructure methods of beam forming in a number of ways [174]–[178].

An array using a weight vector contained in the signal space such that it is orthogonal to the interference-direction steering vector is able to cancel the interference. In situations where the directions of interferences are not known, the weight is estimated by minimizing a suitably selected cost function. A weight estimation method that minimizes a cost function applicable to a digital communications system using a BPSK modulating scheme discussed in [176] demonstrates the utility of this beam-forming concept.

An application of the eigenstructure method for estimating weights of beam-space processors using eigenvectors of the R_N, that is, the matrix with the signal component removed, as is done for secondary beams, suggests the effectiveness of this method for interference canceling [178], [179] in beam space and for achieving the desired performance in a short observation time. An application of the eigenstructure method for correcting errors in steering vectors is reported in [174].

Forming beams using eigenvectors associated with the largest eigenvalues of R for mobile communications applications has been reported in [180].

III. ADAPTIVE BEAM FORMING

In practice, neither R nor R_N is available to calculate the optimal weights of the array, and the weights are adjusted by some means using the available information derived from the array output, array signals, and so on to make an estimate of the optimal weights. There are many such schemes, which are normally referred to adaptive algorithms. Some of these algorithms are described here, and their characteristics, such as the speed of adaption and the mean and variance of the estimated weights, and the parameters affecting these characteristics are briefly discussed.

A. SMI Algorithm

This algorithm estimates the array weights by replacing R with its estimate. An unbiased estimate of R using N samples $\underline{x}(n)$, $n = 0, 1, 2, \cdots, N - 1$ of the array signals may be obtained using a simple averaging scheme

$$R(n) = \frac{1}{N} \sum_{n=0}^{N-1} \underline{x}(n)\underline{x}^H(n) \qquad (85)$$

where $R(n)$ denotes the estimate at the nth instant of time and $\underline{x}(n)$ denotes the array signal sample, also known as the array snapshot, at the nth instant of time, with t replaced by nT and the sampling time T omitted for the ease of notation.

The estimate of R may be updated when the new samples arrive using

$$R(n+1) = \frac{nR(n) + \underline{x}(n+1)\underline{x}^H(n+1)}{n+1} \qquad (86)$$

and a new estimate of the weights $\underline{w}(n+1)$ at time instant $n+1$ may be made. The expression of the optimal weights requires the inverse of R, and this process of estimating R and then its inverse may be combined to update the inverse of R from array signal samples using the Matrix Inversion Lemma as follows:

$$R^{-1}(n) = R^{-1}(n-1) \\ - \frac{R^{-1}(n-1)\underline{x}(n)\underline{x}^H(n)R^{-1}(n-1)}{1+\underline{x}^H(n)R^{-1}(n-1)\underline{x}(n)} \qquad (87)$$

with

$$R^{-1}(0) = \frac{1}{\varepsilon_0}I, \qquad \varepsilon_0 > 0. \qquad (88)$$

This scheme of estimating weights using the inverse update is referred to as the RLS algorithm, which is further described in Section III-C.

It should be noted that as the number of samples grows, the matrix update approaches its true value, and thus the estimated weights approach the optimal weights, that is, as $n \rightarrow \infty$, $R(n) \rightarrow R$ and $\underline{w}(n) \rightarrow \hat{\underline{w}}$ or $\hat{\underline{w}}_{\text{MSE}}$, as the case may be. More discussion on the SMI algorithm may be found in [40] and [181]. Procedures for estimating array weights with efficient computation using SMI are considered in [182], and an analysis to show how it

performs as a function of the number of snapshots is provided in [89].

Application of SMI to estimate the weights of an array to operate in mobile communications systems has been considered in many studies [56], [59], [60], [183]–[186]. The study in [183] considers beam forming for GSM signals using a variable reference signal as available during the symbol interval of the TDMA system. An application discussed in [184] is for vehicular mobile communications, whereas that presented in [186] is for inducing delay spread in indoor radio channels. A presentation in [59] is for mobile satellite communications systems.

B. LMS Algorithm

The application of the LMS algorithm to estimate the optimal weights of an array is widespread, and its study has been of considerable interest for some time now. The algorithm is referred to as the constrained LMS algorithm when the weights are subjected to constraints at each iteration. It is referred to as an unconstrained LMS algorithm when the weights are not constrained at each iteration. The latter is mostly applicable when weights are updated using a reference signal and no knowledge of the direction of the signal is utilized, as is the case for the constrained case.

The algorithm updates the weights at each iteration by estimating the gradient of the quadratic surface and then moving the weights in the negative direction of the gradient by a small amount. The constant that determines this amount is normally referred to as the step size. When this step size is small enough, the process leads these estimated weights to the optimal weights. The convergence and the transient behavior of these weights, along with their covariance, characterize the LMS algorithm, and the way that the step size and the process of gradient estimation affect these parameters is of great practical importance. These and other issues are now discussed in detail.

1) Unconstrained LMS Algorithm: A real-time unconstrained LMS algorithm for determining optimal weight $\hat{\underline{w}}_{\mathrm{MSE}}$ of the system using the reference signal is [27], [187]–[199]

$$\underline{w}(n+1) = \underline{w}(n) - \mu g(\underline{w}(n)) \qquad (89)$$

where $\underline{w}(n+1)$ denotes the new weights computed at the $(n+1)$th iteration; μ is a positive scalar (gradient step size) that controls the convergence characteristic of the algorithm, that is, how fast and how close the estimated weights approach the optimal weights; and $g(\underline{w}(n))$ is an unbiased estimate of the gradient of the MSE

$$\begin{aligned} \mathrm{MSE}(\underline{w}(n)) =& E[|r(n+1)|^2] \\ &+ \underline{w}^H(n)R\underline{w}(n) - 2\underline{w}^H(n)\underline{z} \end{aligned} \qquad (90)$$

at the nth iteration with respect to $\underline{w}(n)$, given by

$$\nabla_{\underline{w}}\mathrm{MSE}(\underline{w})|_{\underline{w}=\underline{w}(n)} = 2R\underline{w}(n) - 2\underline{z}. \qquad (91)$$

It should be noted that at the $(n+1)$th iteration, the array is operating with weights $\underline{w}(n)$ computed at the previous iteration. The array signal vector, however, is $\underline{x}(n+1)$, the reference signal sample is $r(n+1)$, and the array output

$$y(n) = \underline{w}^H(n)\underline{x}(n+1). \qquad (92)$$

In its standard form, the LMS algorithm uses an estimate of the gradient by replacing R and $\underline{z}$ with their noisy estimates available at the $(n+1)$th iteration, leading to

$$\begin{aligned} g(\underline{w}(n)) =& 2\underline{x}(n+1)\underline{x}^H(n+1)\underline{w}(n) \\ &- 2\underline{x}(n+1)r(n+1) \\ =& 2\underline{x}(n+1)\varepsilon^*(\underline{w}(n)) \end{aligned} \qquad (93)$$

where $\varepsilon(\underline{w}(n))$ is the error between the array output and the reference signal, that is

$$\varepsilon(\underline{w}(n)) = \underline{w}^H(n)\underline{x}(n+1) - r(n+1). \qquad (94)$$

Thus, the estimated gradient is a product of the error between the array output and the reference signal as well as the array signals after the nth iteration. For $\mu < 1/\lambda_{\max}$, with $\lambda_{\max}$ denoting the maximum eigenvalue of R, the algorithm is stable and the mean value of the estimated weights converges to the optimal weights. As the sum of all eigenvalues of R equals its trace, the sum of its diagonal elements, one may select the gradient step size μ in terms of measurable quantities using $\mu < 1/\mathrm{Tr}(R)$, with $\mathrm{Tr}(R)$ denoting the trace of R. It should be noted that each diagonal element of R is equal to the average power measured on the corresponding element of the array. Thus, for an array of identical elements, the trace of R equals the power measured on any one element times the number of elements in the array.

The convergence speed of the algorithm refers to the speed by which the mean of the estimated weights (ensemble average of many trials) approaches the optimal weights. It normally is characterized by L trajectories along L eigenvectors of R with the time constant of the ℓth trajectory given by

$$\tau_\ell = \frac{1}{2\mu\lambda_\ell} \qquad (95)$$

with λ_ℓ denoting the ℓth eigenvalue of R. Thus, these time constants are functions of the eigenvalues of R, the smallest one dependent upon $\lambda_{\max}$, which normally corresponds to the strongest source, and the largest one controlled by the smallest eigenvalue, which corresponds to the weakest source or the background noise. Therefore, the larger the eigenvalue spread, the longer it takes for the algorithm to converge. In terms of interference rejection capability, this means canceling the strongest source first and the weakest source last.

The convergence speed of an algorithm is an important property, and its importance for mobile communications is highlighted in [200] by discussing how the LMS algorithm does not perform as well as some other algorithms due to its slow convergence speed in situations of fast-changing signal characteristics. The availability of time for an algorithm to converge in mobile communications systems depends not only on the system design, which dictates the duration of

the user signal present (such as the user slot duration in a TDMA system) but also on the speed of mobiles, which changes the rate at which a signal fads. For example, a mobile on foot would cause the signal to fade at a rate of about 5 Hz, whereas the rate would be on the order of about 50 Hz for a vehicle mobile, implying that an algorithm needs to converge faster in a system being used by vehicle mobiles compared to one used by a hand-held portable device [47]. Some of these issues for an IS-54 system are discussed in [56], where the convergence of the LMS and SMI algorithms in mobile communications situations is compared.

Even when the mean of the estimated weights converges to the optimal weights, they have finite covariance, that is, their covariance matrix is not identical to a matrix with all its elements equal to zero. The covariance matrix of the weights is defined as

$$k_{\underline{w}\,\underline{w}}(n) = E[(\underline{w}(n) - \bar{\underline{w}})(\underline{w}(n) - \bar{\underline{w}})] \qquad (96)$$

where $\bar{\underline{w}} = E[\underline{w}(n)]$ denotes the mean of the estimated weights at the nth iteration. This causes the average of the MSE not to converge to the MMSE and leads to the excess MSE. From the expressions of the MSE and MMSE, it follows that for a given $\underline{w}(n)$, the MSE is given by

$$\text{MSE}(\underline{w}(n)) = \text{MMSE} + \underline{V}^H(n)R\underline{V}(n) \qquad (97)$$

where

$$\underline{V}(n) = \underline{w}(n) - \hat{\underline{w}} \qquad (98)$$

is the difference between the estimated weights and the optimal weights at the nth iteration. Note that $E[\underline{V}(n)] \to 0$ as $n \to \infty$. As all elements of $k_{\underline{w}\underline{w}}(n)$ do not approach zero as $n \to \infty$, it follows that the average value of the excess MSE does not approach zero as $n \to \infty$, that is, $\lim_{n\to\infty} E[\underline{V}^H(n)R\underline{V}(n)] \neq 0$.

The transient and steady-state behavior of the weight covariance matrix and the average excess MSE are important parameters of the LMS algorithm and are discussed in detail in [188] and [198]. A study of the convergence of the LMS algorithm applicable to the PIC processor and a discussion on the gradient step size selection can be found in [75].

The difference between the weights estimated by the adaptive algorithm and the optimal weights is further characterized by the ratio of the average excess steady-state MSE and the MMSE. It is referred as the misadjustment. It is a dimensionless parameter that measures the performance of the algorithm. The misadjustment is a kind of noise and is caused by the use of the noisy estimate of the gradient. This noise is referred to as the misadjustment noise. For the present case when the gradient is estimated by multiplying the array signals with the error between the array output and the reference signal and the gradient step size is selected such that

$$0 < \mu < \frac{1}{4\lambda_{\max}} \qquad (99)$$

and

$$\eta(\mu) \triangleq \sum_{i=1}^{L} \frac{2\mu\lambda_i}{1 - 2\mu\lambda_i} < 1 \qquad (100)$$

then the misadjustment M is given by

$$M = \frac{\eta(\mu)}{1 - \eta(\mu)}. \qquad (101)$$

For a sufficiently small μ, this results in $M \approx 2\mu \text{Tr}(R)$.

It follows from this expression that increasing μ increases the misadjustment noise. On the other hand, an increase in μ causes the algorithm to converge faster, as discussed earlier. Thus, the selection of the gradient step size requires satisfying conflicting demands of 1) reaching vicinity of the solution point more quickly but wandering around over a larger region and causing a bigger misadjustment and 2) arriving near the solution point slowly with the smaller movement in the weights at the end. The latter causes an additional problem, particularly in a nonstationary environment, say, when the interference and optimal solution move slowly, causing adapting estimated weights to lag behind the optimal weights. This phenomenon is referred to as the weight vector lag.

Many schemes, including variable step size, have been suggested to overcome this problem [201]–[208]. Some of these schemes are now discussed.

The adaptive algorithm estimates the weights by minimizing the MSE. Thus, in schemes where a variable step size is used, it reflects the value of the MSE at that iteration (going up and down as the MSE goes up and down) such that it stays between the maximum permissible value for convergence and the minimum value based upon the allowed misadjustment. It may be truly variable or it may be allowed to switch between a few preselected values for the ease of implementation, as well as to shift by one bit left or right where digital implementation is used. The step size may also be adjusted to reflect the change in the direction of the gradient of error surface at each iteration [207].

The optimal value of the step size at each step is suggested in [203] such that it minimizes the MSE at each iteration. This is a function of the value of the true gradient at each iteration and R. In practice, these may be replaced by their instantaneous values, leading to a suboptimal value.

Instead of having a single step size for an entire weight vector, one may select a variable step size for each weight separately, leading to an increased convergence of the algorithm [204]. The convergence speed of an algorithm may also be increased by adjusting the weights such that interferences are canceled one at a time [209], [210] and by using a scheme known as block processing [211]. For broad-band signals, an implementation in the frequency domain may help increase the speed of convergence.

The application of frequency-domain beam forming to estimate the weights using the LMS algorithm for the case when a reference signal is available [138], [139], [142], [143] shows how the frequency-domain approach yields improved convergence and reduced computational
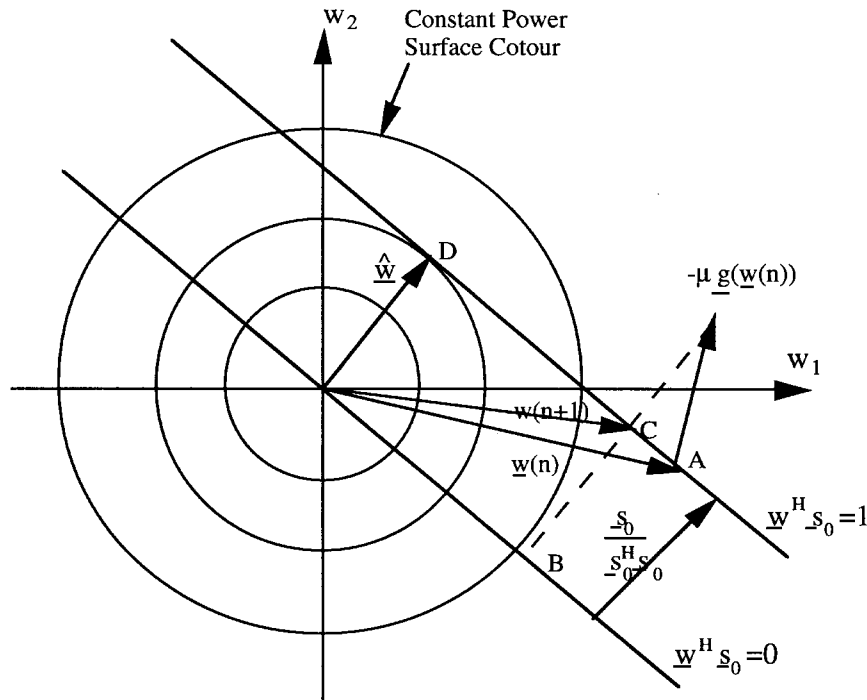
Fig. 12. Constrained LMS algorithm: pictorial view of the projection process.

complexities over the time-domain approach. Improved convergence normally arises from the use of different gradient step sizes in different bins. For the constrained LMS case, this is likely to cause deterioration in the steady-state performance of the algorithm. This deterioration, however, does not affect the performance of the unconstrained algorithm [212].

An algorithm known as a sign algorithm [208], [213], where the error between the array output and the reference signal is replaced by its sign, is computationally less complex than the LMS algorithm, as discussed.

The algorithm is usually analyzed assuming that successive samples are uncorrelated. This assumption helps in simplifying the mathematics by allowing expectations of data products to be replaced by the products of their expectations. A discussion of situations of correlated samples and a nonstationary environment may be found in [214]–[216].

Applications of an unconstrained LMS algorithm to mobile communications systems using an array include base-mobile communications systems [46], indoor-radio systems [47], and satellite-to-satellite communications systems [97].

2) Normalized LMS Algorithm: This algorithm is a variation of the constant-step-size LMS algorithm and uses a data-dependent step size at each iteration. At the nth iteration, the step size is given by

$$\mu(n) = \frac{\mu_0}{\underline{x}^H(n)\underline{x}(n)} \quad (102)$$

where μ_0 is a constant. The algorithm and its convergence using various types of data have been studied widely [217]–[224]. It avoids the need for estimating the eigenvalues of the correlation matrix or its trace for selection of the maximum permissible step size. The algorithm normally has better convergence performance and less

signal sensitivity compared to the normal LMS algorithm. A discussion of its application to mobile communications can be found in [225].

3) Constrained LMS Algorithm: A real-time constrained algorithm [7], [25], [226]–[233] for determining the optimal weight vector $\hat{\underline{w}}$ is

$$\underline{w}(n+1) = P\{\underline{w}(n) - \mu\underline{g}(\underline{w}(n))\} + \frac{\underline{s}_0}{\underline{s}_0^H \underline{s}_0} \quad (103)$$

where

$$P \triangleq I - \frac{\underline{s}_0 \underline{s}_0^H}{L} \quad (104)$$

is a projection operator, $\underline{g}(\underline{w}(n))$ is an unbiased estimate of the gradient of the power surface $\underline{w}^H(n)R\underline{w}(n)$ with respect to $\underline{w}(n)$ after the nth iteration, μ is the gradient step size, and $\underline{s}_0$ is the steering vector in the look direction.

The algorithm is "constrained" because the weight vector satisfies the constraint at every iteration, that is, $\underline{w}^H(n)\underline{s}_0 = 1$, $\forall n$. The process of imposing constraints may be understood from Fig. 12, which shows how weights are updated and how a projection system uses a vector diagram for a two-weight system [25]. The figure shows constant power contours, the constraint surface (a line $\underline{w}^H \underline{s}_0 = 1$ for a two-dimensional system), a surface parallel to the constraint surface passing through the origin ($\underline{w}^H \underline{s}_0 = 0$), weight vectors $\underline{w}(n)$, $\underline{w}(n+1)$, and $\hat{\underline{w}}$, and the gradient at the nth iteration.

The point A on the diagram indicates the position of the weight after completion of the nth iteration. It is the cross section of the constraint equation $\underline{w}^H \underline{s}_0 = 1$ and the power surface $\underline{w}^H(n)R\underline{w}(n)$ (not shown in the figure). The weights are perturbed by adding a small amount $-\mu\underline{g}(\underline{w}(n))$ and then are projected on $\underline{w}^H \underline{s}_0 = 0$ using

projection operator P. This point is indicated by B on the diagram. Note that $P\underline{s}_0 = 0$. Thus, the projection operator projects the weights orthogonal to $\underline{s}_0$. The constraint now is restored by adding $\underline{s}_0/\underline{s}_0^H \underline{s}_0$ and the updated weights $\underline{w}(n+1)$ move to point C. The process continues by moving the estimated weights toward point D, the optimal solution.

The effect of the gradient step size μ on the convergence speed and the misadjustment noise may also be understood using this figure. A larger step size means that the weight vector moves faster toward point D, the solution point, but wanders around it over a larger region, not reaching close to it and causing more misadjustment.

The gradient of $\underline{w}^H(n)R\underline{w}(n)$ with respect to $\underline{w}(n)$ is given by

$$g(\underline{w}(n)) \triangleq \nabla_{\underline{w}}(\underline{w}^H R \underline{w})|_{\underline{w}=\underline{w}(n)} = 2R\underline{w}(n) \qquad (105)$$

and its computation using this expression requires knowledge of R, which normally is not available in practice. For a standard LMS algorithm, an estimate of the gradient at each iteration is made by replacing R by its noisy sample $\underline{x}(n+1)\underline{x}^H(n+1)$ available at time instant $(n+1)$, leading to $g(\underline{w}(n)) = 2\underline{x}(n+1)\,y^*(\underline{w}(n))$.

Thus, the gradient estimate is the product of the array signals and the array output available after the nth iteration. The mean value of the weights estimated by the algorithm using this gradient converges to the optimal weights, provided that the gradient step size is small enough to satisfy

$$0 < \mu < \frac{1}{2\lambda_{\max}(PRP)}. \qquad (106)$$

The convergence of the mean weights to $\underline{\hat{w}}$ along the ℓth eigenvector of PRP has the time constant

$$\hat{\tau}_\ell = \frac{-1}{\ell n[1 - 2\mu\lambda_\ell(PRP)]} \approx \frac{1}{2\mu\lambda_\ell(PRP)} \qquad (107)$$

where $\ell n[\cdot]$ denotes the natural logarithm of $[\cdot]$ and λ_ℓ (PRP) and $\lambda_{\max}$ (PRP), respectively, denote the ℓth eigenvalue and the maximum eigenvalue of PRP.

It follows from

$$R = p_s \underline{s}_0 \underline{s}_0^H + R_N \qquad (108)$$

and

$$P\underline{s}_0 = 0 \qquad (109)$$

that $PRP = PR_N P$, and hence the convergence speed of the mean value of the weights characterized by the time constants and the upper limit on the gradient step size depends only on the eigenvalues of $PR_N P$, indicating that the signal arriving from the look direction does not affect these quantities. The eigenvalues of $PR_N P$ are a function of the directions and powers of the directional sources as well as the array geometry, with the maximum eigenvalue controlled by the strongest source governing the initial convergence speed. The latter part of the convergence is controlled by the smaller eigenvalues associated with the weak sources or the background noise, and thus the overall speed of the algorithm depends upon the eigenvalue spread of $PR_N P$.

The discussion so far has concentrated on the convergence of the mean value of the weights to the optimal weights. The variance of these weights is an important parameter, and the transient and steady-state behavior of the weight covariance matrix $k_{\underline{w}\,\underline{w}}(n)$ are indicators of the performance of the algorithm, as discussed previously for the unconstrained LMS algorithm. An expression for $k_{\underline{w}\,\underline{w}}(n)$ indicates [228] that it is a function of the variance of the gradient estimate. For the standard algorithm, an expression for the variance of the gradient is given by

$$V_g(\underline{w}(n)) = 4\underline{w}^H(n)R\underline{w}(n)R. \qquad (110)$$

The steady-state value of the weight covariance matrix governs the misadjustment. For the standard algorithm, it is given by

$$M = \frac{\mu \sum_{i=1}^{L-1} \frac{1}{1-\mu\lambda_i(PRP)}}{1 - \mu \sum_{i=1}^{L-1} \frac{1}{1-\mu\lambda_i(PRP)}}. \qquad (111)$$

a) Signal sensitivity: The convergence of the mean weights to the optimal weights is a function of the eigenvalues of $PR_N P$ and thus is independent of the look direction signal. This is not the case, however, for the weight covariance matrix, which depends on the projected covariance of the gradient used for the weight update algorithm, that is, $PV_g(\underline{w}(n))P$. For the standard algorithm, this variance is a product of R and the mean output power $\underline{w}^H(n)R\underline{w}(n)$ at the nth instant of time. Thus, $PV_g(\underline{w}(n))P$, which is proportional to $\underline{w}^H(n)R\underline{w}(n)PRP$, contains a signal from the look direction indicating that the performance of the standard LMS algorithm is not independent of the signal and that the transient behavior of weight covariance depends on it. The following, a rather heuristic argument, explains how the signal level causes the weights to fluctuate using an explicit expression of weights rather than their weight covariance matrix. Rewrite the constrained LMS algorithm as follows:

$$\underline{w}(n+1) = P\underline{w}(n) + \frac{\underline{s}_0}{\underline{s}_0^H \underline{s}_0} - \mu P g(\underline{w}(n)) \qquad (112)$$

and examine the projected gradient vector term $Pg(\underline{w}(n))$.

Expressing

$$g(\underline{w}(n)) = \underline{x}(n+1)\underline{x}^H(n+1)\underline{w}(n) \qquad (113)$$

and noting that

$$\underline{x}(n) = m_s(n)\underline{s}_0 + \underline{x}_N(n) \qquad (114)$$

with $m_s(n)$ denoting the sample of the complex modulating function of the signal and $\underline{x}_N(n)$ being the array receiver vector not containing the signal, one obtains

$$Pg(\underline{w}(n)) = P\underline{x}_N(n+1)x_N^H(n+1)\underline{w}(n)$$
$$+ m_s^*(n+1)Px_N(n+1)\underline{s}_0^H \underline{w}(n). \qquad (115)$$

$m_s^*(n+1)$ is a random quantity with variance equal to the look-direction signal power. This makes $Pg(\underline{w}(n))$ a noisy

quantity that fluctuates with the signal power and causes the $\underline{w}(n+1)$ to fluctuate. The fluctuations in $\underline{w}(n+1)$ increase as the signal power increases. Thus, the weights estimated by the standard algorithm are sensitive to the signal power, requiring a lower step size in the presence of a strong signal for the algorithm to converge, which in turn reduces its convergence speed.

This fact has been demonstrated in [234] for a high-speed GMSK mobile communications system. The system has been implemented by mounting an array on a vehicle to measure its BER performance.

The signal sensitivity of the standard LMS algorithm is caused by the use of a sample correlation matrix in estimating the gradient and could be reduced by using an estimate of the correlation matrix from all available samples. A recursive LMS algorithm uses all previous samples and updates the correlation matrix as a new sample arrives, using

$$R(n+1) = \frac{nR(n) + \underline{x}_N(n+1)\underline{x}_N^H(n+1)}{n+1}. \quad (116)$$

The algorithm then uses this matrix to estimate the required gradient

$$\underline{g}(\underline{w}(n)) = 2R(n+1)\underline{w}(n). \quad (117)$$

The estimated gradient is unbiased and has variance

$$V\underline{g}(\underline{w}(n)) = \frac{4}{(n+1)^2}\underline{w}^H(n)R\underline{w}(n)R. \quad (118)$$

Comparing this with the variance of the standard LMS algorithm, it follows that the variance of the gradient was reduced by a factor of $(n+1)^2$ using the recursive method, thus making the recursive algorithm less signal sensitive. As $n \rightarrow \infty$, the signal sensitivity of the recursive LMS algorithm approaches zero.

The signal sensitivity of the LMS also can be reduced by spatial averaging instead of sample averaging, as is done when the weights are estimated using a structured gradient algorithm.

b) Structured gradient algorithm: For a linear array of equispaced elements, the array correlation matrix has the Toeplitz structure, that is

$$R \equiv \begin{pmatrix} r_0 & r_1 & \cdots & & r_{L-1} \\ r_H^* & \cdot & & & \\ \cdot & & \cdot & & \\ \cdot & & & \cdot & \\ r_{L-1}^* & & & & r_0 \end{pmatrix} \quad (119)$$

with r_i, $i = 0, 1, \cdots, L-1$ being the L correlation lags. The noisy sample of R used in estimating the gradient for the standard LMS algorithm does not have this structure. The structured gradient algorithm [231], [232] exploits this structure of R such that the estimated matrix has this structure. The ith lag $r_i(n)$ is estimated as

$$r_i(n) = \frac{1}{N_i}\sum_\ell x_\ell(n)x_{\ell+i}^*(n), \qquad i = 0, 1, \cdots, L-1 \quad (120)$$

where N_i denotes the number of possible combinations of elements with lag i and summation is over all these combinations. For a linear array of equispaced elements, $N_i = L - i$. It should be noted that for a nonuniform linear array, the amount of improvement realized by the structured method would depend upon the number of elements in R with the same correlation lag.

An algorithm that uses the structured method to estimate the matrix using all available samples is discussed in [233]. It has a better convergence performance than that of the RLS algorithm in the presence of a strong look-direction signal. The algorithm is referred to as the improved LMS algorithm.

The discussion of the LMS algorithm implies that one has access to all array signals. In situations where this access is not available or not economical, one could estimate the required gradient using perturbation schemes [226]–[228], [235]. Algorithms using these schemes perturb the array weights using some orthogonal sequences and use the measured array output power over the perturbation cycle to estimate the gradient. For a perturbation cycle of length J, for example, the algorithm requires J samples at each iteration to estimate the gradient. Thus, the iteration number and the sample numbers are different and the algorithm is slower by a factor of J when measured in time rather than iteration number. The gradient estimation also adds additional noise to the system, known as the perturbation noise.

A method similar to that used in [236] for adjusting equalizer taps can also be used for adjusting array weights. The method uses a running average of the past gradients to estimate the required gradient at the nth iteration rather than using the past correlation matrixes to estimate $R(n)$, as is done in the recursive LMS case to reduce the weight noise.

It should be noted that all of these gradient estimating schemes—which reduce the variance of the gradient, leading to less fluctuations in array weights, inherently increase the convergence speed of the algorithm as one is able to increase the step size without compromising the stability of the algorithm.

3) Implementation Issues: The convergence speed, fluctuations in array weights during adaption, and misadjustment noise are the measures of the transient and steady-state behavior of the LMS algorithm. The theoretical performance of the algorithm and the effect of the look-direction signal and gradient step size discussed in the previous section assume the existence of infinite precision, that is, the variable is allowed to take any value. Now, the implications of finite-precision implementations are briefly discussed.

a) Finite-precision arithmetic: In real life, when the algorithm is implemented using digital hardware, where a variable can take only discrete values, there are other parameters that affect its performance and other issues that need consideration, including quantization noise as well as roundoff and truncation noise caused by finite-precision arithmetic [204], [237]–[244].

First, when a b-bit quantizer is used to convert an analog signal of range $-r_{\max}$ to $r_{\max}$ into a digital signal, it adds

a quantization noise of zero mean and variance [245]

$$\sigma_q^2 = \frac{2^{-2b} r_{\max}^2}{3} \tag{121}$$

to the system. Second, the effect of the finite word length of the devices where the numbers are stored causes the roundoff or truncation noise to be added to the system. This arises from the fact that when arithmetic operations are performed using these numbers, the answers are normally longer than the available word length and thus need to be rounded off or truncated to fit into finite word memory. Last, all the variables, such as the estimated gradient, gradient step size, and estimated weights, are allowed to take only finite values and can be increased or decreased by a factor of two. The combined effect of all these on the algorithm is a larger fluctuation in the weights and a larger misadjustment than otherwise.

The misadjustment appears to be the most sensitive to the finite word length effect on weights, suggesting that the weights should be implemented using a longer word length [237]. A reduction in the step size below certain levels may even cause the misadjustment to increase [242], which is contrary to the infinite-precision case, where a decrease in the step causes the misadjustment to decrease. It appears [244] that the finite word length effects are amplified in an environment that yields smaller eigenvalues for the correlation matrix.

An important effect of the finite word length on the weight update is that when a small input does not cause the weights to move more than the least significant bit (the smallest possible increment, which depends upon the number of bits used to store weights), then the algorithm stalls and the weights do not change anymore [242], requiring a bigger step size, which in turn increases the weight fluctuations.

A postalgorithm smoothing scheme suggested in [238] appears to reduce the weight fluctuations, leading to a better convergence performance. It suggests a running average of past weights. Thus, the weights are recursively updated using past weights with or without finite memory.

A discussion of system design applicable to mobile satellite communications that takes into account quantization noise and other issues discussed above may be found in [59].

b) Real versus complex implementation: There are situations where the input data to the weight adaption scheme are real, and situations where these are complex (with real and imaginary parts denoting in-phase and quadrature components). In both of these cases, the weights could be updated using the real LMS algorithm or the complex LMS algorithm. The former utilizes real arithmetic and uses real variables and updates real weights (the in-phase and quadrature components are updated separately when complex data are available), whereas the complex algorithm [246] utilizes complex arithmetic, uses complex variables, and updates as well as implements weights as complex variables similar to the treatment presented in this paper. For real data using a complex algorithm, one needs to generate the quadrature

component using the Hilbert transformer or quadrature filter [247], which has the transfer functions

$$H(f) = \begin{cases} -j & f > 0 \\ j & f < 0. \end{cases} \tag{122}$$

For a similar misadjustment, the complex algorithm converges faster than the real algorithm. For more details on this aspect, see, for example, [198] and [228].

C. RLS Algorithm

The convergence of the LMS algorithm depends upon the eigenvalues of R. In an environment yielding R with a large eigenvalue spread, the algorithm converges with slow speed. This problem is solved in an RLS algorithm [64], [248]–[258] by replacing the gradient step size μ with a gain matrix $R^{-1}(n)$ at the nth iteration, producing the weight update equation

$$\underline{w}(n) = \underline{w}(n-1) - R^{-1}(n)\underline{x}(n)\varepsilon^*(\underline{w}(n-1)) \tag{123}$$

where $R(n)$ is given by

$$R(n) = \delta_0 R(n-1) + \underline{x}(n)\underline{x}^H(n)$$
$$= \sum_{k=0}^{n} \delta_0^{n-k} \underline{x}(k)\underline{x}^H(k) \tag{124}$$

where δ_0, a real scalar smaller than but close to one, is used for exponential weighting of the past data and is referred to as the forgetting factor, as the update equation tends to deemphasize the old samples. The quantity $1/1 - \delta_0$ is normally referred to as the memory of the algorithm. Thus, for $\delta_0 = .99$, the memory of the algorithm is close to 100 samples. The RLS algorithm updates the required inverse of $R(n)$ using the previous inverse and the present sample as

$$R^{-1}(n) = \frac{1}{\delta_0}\left[R^{-1}(n-1) \right.$$
$$\left. - \frac{R^{-1}(n-1)\underline{x}(n)\underline{x}^H(n)R^{-1}(n-1)}{\delta_0 + \underline{x}^H(n)R^{-1}(n-1)\underline{x}(n)} \right]. \tag{125}$$

The matrix is initialized as

$$R^{-1}(0) = \frac{1}{\varepsilon_0}I, \qquad \varepsilon_0 > 0. \tag{126}$$

A discussion on the selection of ε_0 and its effects on the performance of the algorithm can be found in [253]. The RLS algorithm minimizes the cumulative square error [251], [252]

$$J(n) = \sum_{k=0}^{n} \delta_0^{n-k} |\varepsilon(k)|^2 \tag{127}$$

and its convergence is independent of the eigenvalue distribution of the correlation matrix.

The algorithm presented here is the exact RLS algorithm. For other forms of the RLS algorithm with improved computation efficiency, see, for example, [249] and [253]. A comparison of the convergence speed of the LMS, the RLS, and some other gradient-based algorithms using

quantized or clipped data indicates that RLS is the most efficient and LMS is the slowest [259].

A computer-simulation study of the RLS, LMS, and SMI algorithms in a mobile communications situation suggests that the former outperforms the latter two in flat-fading channels [260]. An application of the RLS algorithm for the reverse link of a cellular communication using the CDMA system is considered in [261] to show an increase in channel capacity by an adaptive array.

D. CMA

CMA is a gradient-based algorithm that works on the premise that the existence of an interference causes fluctuation in the amplitude of the array output, which otherwise has a constant modulus. It updates the weights by minimizing the cost function [96], [262]–[264]

$$J(n) = \frac{1}{2} E\big[\big(|y(n)|^2 - y_0^2\big)^2\big] \qquad (128)$$

using the following equation:

$$\underline{w}(n+1) = \underline{w}(n) - \mu \underline{g}(\underline{w}(n)) \qquad (129)$$

where $y(n) = \underline{w}^H(n)\underline{x}(n+1)$ is the array output after the nth iteration, y_0 is the desired amplitude in the absence of interference, and $\underline{g}(\underline{w}(n))$ denotes an estimate of the gradient of the cost function. Similar to the LMS algorithm discussed previously, it uses an estimate of the gradient by replacing the true gradient with an instant value given by

$$\underline{g}(\underline{w}(n)) = 2\varepsilon(n)\underline{x}(n+1) \qquad (130)$$

where

$$\varepsilon(n) \triangleq \big(|y(n)|^2 - y_0^2\big)y(n). \qquad (131)$$

The weight update equation for this case becomes

$$\underline{w}(n+1) = \underline{w}(n) - 2\mu\varepsilon(n)\underline{x}(n+1). \qquad (132)$$

In appearance, this is similar to the LMS algorithm with a reference signal where

$$\varepsilon(n) \triangleq d(n) - y(n). \qquad (133)$$

Its application to a digital land-mobile radio communications system using TDMA is studied in [265] to compensate for selective fading. Discussions of hardware implementation of a CMA adaptive array and its BER performance for high-speed transmission in mobile communications may be found in [234] and [266]. Development of CMA for beam-space array signal processing, including its hardware realization, has been reported in [99]. The results presented in [96] indicate that the beam-space CMA is able to cancel interferences arriving from directions other than the look direction.

CMA is useful for eliminating correlated arrivals and is effective for constant modulated envelope signals such as GMSK and QPSK, which are used in digital communications. The algorithm, however, is not appropriate for the CDMA system because of the required power control [261]. Use of CMA to separate cochannel FM signals blindly

in mobile communications has been investigated in [267]. A variation of CMA referred to as differential CMA and reported in [180] has inferior convergence characteristics compared to CMA but may be improved using DOA information to make it operative in beam space.

E. Conjugate Gradient Method

An application of the conjugate gradient method [268]–[270] to adjust the weights of an antenna array is discussed in [57] and [271]. The method in general is useful for solving a set of equations of the form $A\underline{w} = \underline{b}$ to obtain $\underline{w}$.

For an array processing problem [57], [271], $\underline{w}$ denotes the array weights, A is a matrix with each of its columns denoting consecutive samples obtained from array elements, and $\underline{b}$ is a vector containing consecutive samples of the desired signal. Thus, a residual vector

$$\underline{r} = \underline{b} - A\underline{w} \qquad (134)$$

denotes an error between the desired signal and the array output at each sample, with the sum of the squared error given by $\underline{r}^H\underline{r}$.

The method starts with an initial guess $\underline{w}(0)$ of the weights, obtains a residual

$$\underline{r}(0) = \underline{b} - A\underline{w}(0) \qquad (135)$$

and an initial direction vector

$$\underline{g}(0) = A^H\underline{r}(0) \qquad (136)$$

and moves the weights in this direction to yield a weight update equation

$$\underline{w}(n+1) = \underline{w}(n) - \mu(n)\underline{g}(n) \qquad (137)$$

where the step size

$$\mu(n) \triangleq \frac{|A^H\underline{r}(n)|^2}{|A^H\underline{g}(n)|^2}. \qquad (138)$$

The residual $\underline{r}(n)$ and the direction vector $\underline{g}(n)$ are updated using

$$\underline{r}(n+1) = \underline{r}(n) + \mu(n)A\underline{g}(n) \qquad (139)$$

and

$$\underline{g}(n+1) = A^H\underline{r}(n+1) - \alpha(n)\underline{g}(n) \qquad (140)$$

with

$$\alpha(n) \triangleq \frac{|A^H\underline{r}(n+1)|^2}{|A^H\underline{r}(n)|^2}. \qquad (141)$$

The algorithm is stopped when the residual falls below a certain predetermined level. It should be noted that the direction vector points in the direction of the gradient of the error surface $\underline{r}^H(n)\underline{r}(n)$ at the nth iteration, which the algorithm is trying to minimize. The method converges to the minimum of the error surface within at most L iterations for an L-rank matrix equation and thus provides the fastest convergence of all the iterative methods [57], [270], [272].

Use of the conjugate gradient method to eliminate multipath fading in mobile communications situations has been studied in [57] and [271] to show that the BER performance of the system using the conjugate gradient method is better than that using RLS algorithm.

F. Neural Network Approach

In this section, an algorithm referred to as Madaline Rule III (MRIII) is described. A discussion of various aspects of this algorithm as well as other related issues can be found in [273]. For a general theory of neural networks and their applications, see, for example, [274] and [275].

The MRIII algorithm described here is applicable when the reference signal is available and minimizes the MSE between the reference signal and the modified array output rather than the MSE between the reference signal and the array output, as is the case for other algorithms discussed previously. The array output is modified using a nonlinear mapping such as hyperbolic tangent

$$\tanh(x) = \frac{1 - e^{-2x}}{1 + e^{-2x}} \qquad (142)$$

and the weights are updated using

$$\underline{w}(n+1) = \underline{w}(n) - \mu \underline{g}(\underline{w}(n)) \qquad (143)$$

where μ is the gradient step size and $\underline{g}(\underline{w}(n))$ is the instant gradient of the MSE surface with respect to the array weights $\underline{w}(n)$.

When the array is operating with weights $\underline{w}(n)$, producing the array output

$$y(n) = \underline{w}^H(n)\underline{x}(n+1) \qquad (144)$$

the modified output $\tilde{y}(n)$ becomes

$$\tilde{y}(n) = \tanh(y(n)) \qquad (145)$$

and the resulting error signal is given by

$$\tilde{\varepsilon}(n) = \tilde{y}(n) - r(n+1). \qquad (146)$$

The instant gradient of the MSE surface with respect to the array weights $\underline{w}(n)$ thus becomes

$$\begin{aligned}
\underline{g}(\underline{w}(n)) &= \frac{\partial(\tilde{\varepsilon}^*(n)\tilde{\varepsilon}(n))}{\partial \underline{w}(n)} \\
&= 2\tilde{\varepsilon}(n)\frac{\partial \tilde{\varepsilon}(n)}{\partial \underline{w}(n)} \\
&= 2\tilde{\varepsilon}(n)\frac{\partial \tilde{\varepsilon}(n)}{\partial y(n)}\frac{\partial y(n)}{\partial \underline{w}(n)} \\
&= 2\tilde{\varepsilon}(n)\frac{\partial \tilde{\varepsilon}(n)}{\partial y(n)}\underline{x}(n+1).
\end{aligned} \qquad (147)$$

Replacing $\partial \tilde{\varepsilon}(n)/\partial y(n)$ with $\Delta \tilde{\varepsilon}(n)/\Delta y$ for small Δy in (147) results in

$$\underline{g}(\underline{w}(n)) = 2\tilde{\varepsilon}(n)\frac{\Delta \tilde{\varepsilon}(n)}{\Delta y}\underline{x}(n+1) \qquad (148)$$

where $\Delta \tilde{\varepsilon}(n)$ denotes the change in the error output when the array output is perturbed by a small amount Δy and

could be measured to estimate the instant gradient. The weight update equation then becomes

$$\underline{w}(n+1) = \underline{w}(n) - 2\mu\tilde{\varepsilon}(n)\frac{\Delta \tilde{\varepsilon}(n)}{\Delta y}\underline{x}(n+1). \qquad (149)$$

The MSE surface of the error signal $\tilde{\varepsilon}(n)$ may have local minimas, and thus the global convergence of the MRIII algorithm is not guaranteed, which is not the case when MSE between the reference signal and the array output is minimized [273]. The algorithm, however, is very robust, suitable for analog implementation and resulting in fast weight updates.

The MRIII algorithm described here is suitable when the reference signal is available. A scheme to solve a constrained beam-forming problem using neural networks is analyzed in [276], and its implementation using switched capacitor circuits is described in [277]. Computer simulations and experimental results indicate the suitability of the scheme.

IV. DOA ESTIMATION METHODS

In this section, a review of DOA estimation methods, including their performance, sensitivity, and limitations [278], is presented. The direction of a source is parameterized by the variable θ.

A. Spectral Estimation Methods

These methods estimate the DOA by computing the spatial spectrum and then determining the local maximas [43], [279]–[284]. Most of these techniques have their roots in time-series analysis. A brief overview and comparison of some of these methods can be found in [279] and [281].

One of the earliest methods of spectral analysis is the Bartlett method [279], [284], where a rectangular window of uniform weighting is applied to the time-series data to be analyzed. For bearing estimation problems using an array, this is equivalent to equal weighting on each element. Thus, by steering the array in θ direction, this method estimates the mean power, an expression for which is given by

$$P_B(\theta) = \frac{\underline{s}_\theta^H R \underline{s}_\theta}{L^2} \qquad (150)$$

where $\underline{s}_\theta$ denotes the steering vector associated with the direction θ. A set of steering vectors $\{\underline{s}_\theta\}$ associated with different θ is often referred to as the array manifold in DOA estimation literature. In practice, it may be measured at the time of array calibration.

The process is similar to that of mechanically steering the array in this direction and measuring the output power. Due to the resulting side lobes, the output power is not only contributed from the direction in which the array is steered but from the directions where the side lobes are pointing. The processor is also known as the conventional beam former, and the resolving power of the processor depends upon the aperture of the array or the beamwidth of the main lobe. Its use for mobile communications has been studied in [285].

B. MVDR Estimator

This is the ML method of spectrum estimation [43], which finds the ML estimate of the power arriving from a point source in direction θ assuming all other sources as interferences. In beam-forming literature, it is known as the MVDR beam former as well as the optimal beam former since in the absence of errors, it maximizes the output SNR and passes the look-direction signal undistorted. For a DOA estimation problem, the term "maximum likelihood" is used for the method that finds the ML estimate of the direction rather than of the power, as is done by this method [286]. Following this convention, the current estimator in this paper is referred to as the MVDR estimator.

This method uses the array weights, which are obtained by minimizing the mean output power subject to unity constraint in the look direction. An expression for the power spectrum is given by

$$P_{\mathrm{MV}}(\theta) = \frac{1}{\underline{s}_\theta^H R^{-1} \underline{s}_\theta}. \tag{151}$$

This method has better resolution properties than the Bartlett method [42] but does not have the best resolution properties of any method [281].

C. Linear Prediction Method

This method estimates the output of one sensor using linear combinations of the remaining sensor outputs and minimizes the mean square prediction error, that is, the error between the estimate and the actual output [281], [287]. Thus, it obtains the array weights by minimizing the mean output power of the array subject to the constraint that the weight on the selected sensor is unity. An expression for the array weights and the power spectrum is given, respectively, by [281]

$$\hat{\underline{w}} = \frac{R^{-1} \underline{u}_1}{\underline{u}_1^H R^{-1} \underline{u}_1} \tag{152}$$

and

$$P_{\mathrm{LP}}(\theta) = \frac{\underline{u}_1^H R^{-1} \underline{u}_1}{\left| \underline{u}_1^H R^{-1} \underline{s}_\theta \right|^2} \tag{153}$$

where $\underline{u}_1$ is a column vector of all zeros except one element, which is equal to one. The position of one in the column corresponds to the position of the selected element in the array for predicting its output. There is no criterion for proper choice of this element. The choice of this element, however, affects the resolution capability and the bias in the estimate, and these effects are dependent upon the SNR and separation of the directional sources [281]. The linear prediction methods perform well in a moderately low SNR environment and are a good compromise in situations where sources are of approximately equal strength and are nearly coherent [288].

D. MEM

This method finds a power spectrum such that its Fourier transform equals the measured correlation subjected to the constraint that its entropy is maximized [289]. The entropy of a Gaussian band-limited time series with power spectrum $S(f)$ is defined as

$$H(S) = \int_{-f_N}^{f_N} \ln S(f) \, df \tag{154}$$

where f_N is the Nyquist frequency. For estimating DOA from the measurements using an array of sensors, the method finds a continuous function $P_{\mathrm{ME}}(\theta) > 0$ such that it maximizes the entropy function

$$H(P) = \int_0^{2\pi} \ln P_{\mathrm{ME}}(\theta) \, d\theta \tag{155}$$

subject to the constraint that the measured correlation between the ith and the jth element R_{ij} satisfies

$$R_{ij} = \int_0^{2\pi} P_{\mathrm{ME}}(\theta) \cos(2\pi \tau_{ij}(\theta)) \, d\theta \tag{156}$$

where $\tau_{ij}(\theta)$ denotes the differential delay between elements i and j due to a source in θ direction. The solution to this problem requires an infinite dimensional search, which may be transformed to a finite dimensional search using the duality principle [290], leading to

$$P_{\mathrm{ME}}(\theta) = \frac{1}{\hat{\underline{w}}^T \underline{q}(\theta)} \tag{157}$$

where $\hat{\underline{w}}$ is obtained by minimizing

$$H(\underline{w}) = \int_0^{2\pi} \ln(\underline{w}^T \underline{q}(\theta)) \, d\theta \tag{158}$$

subject to

$$\underline{w}^T \underline{r} = 2\pi \tag{159}$$

and

$$\underline{w}^T \underline{q}(\theta) > 0 \quad \forall \theta \tag{160}$$

with $\underline{q}(\theta)$ and $\underline{r}$ defined as

$$\underline{q}(\theta) = [1, \sqrt{2} \cos(2\pi f \tau_{12}(\theta)), \ldots]^T \tag{161}$$

$$\underline{r} = [R_{11}, \sqrt{2} R_{12}, \ldots]^T. \tag{162}$$

It should be noted that the dimension of this vector depends upon the array geometry and is equal to the number of known correlations R_{ij} for every possible i and j.

The minimization problem defined above may be solved iteratively using a standard gradient descent algorithm. More information on various issues of the MEM may be found in [200] and [291]–[295]. The suitability of MEM

for mobile communications in fast-fading signal conditions has been studied in [200].

E. MLM

This method estimates the DOA's from a given set of array samples by maximizing the log-likelihood function [286], [296]–[303]. The likelihood function is the joint probability density function of the sampled data given the DOA's and viewed as a function of the desired variables—the DOA's, for this case. The method searches for those directions that maximize the log of this function, the log-likelihood function. The ML criterion signifies that plane waves from these directions are most likely to cause the given samples to occur [304].

The maximization of the log-likelihood function is a nonlinear optimization problem. In the absence of a closed-form solution, it requires iterative schemes for solutions. There are many such schemes available in the literature. The well-known gradient decent algorithm using the estimated gradient of the function at each iteration as well as the standard Newton–Raphson method are well suited for the job [305]. Other schemes, such as the alternating projection method [298], [300] and the expectation maximization algorithm [286], [306], [307], have been proposed for solving this problem in general as well as for specialized cases, such as unknown polarization [301], unknown noise environments [302], and contaminated Gaussian noise [296]. A fast algorithm [308] based upon Newton's method developed for estimating frequencies of sinusoids may be modified to suit the DOA estimation based upon ML criterion.

The ML method gives a superior performance compared to other methods, particularly when the SNR is small, the number of samples are small, or the sources are correlated [298], and thus is of practical interest. For a single source, the estimates obtained by this method are asymptotically unbiased [301], that is, the expected values of the estimates are equal to their true values. In that sense, it may be used as a standard to compare the performance of other methods. The method normally assumes that the number of sources M are known [298].

When a large number of samples are available, other, computationally more efficient schemes may be used with performance almost equal to this method [299]. Analysis of the method to estimate the direction of sources when the array and the source are in motion relative to each other indicates its potential for mobile communications [309], [310].

F. Eigenstructure Methods

These methods rely on the following properties of R: 1) The space spanned by its eigenvectors may be partitioned into two subspaces, namely, the signal subspace and the noise subspace, and 2) the steering vectors corresponding to the directional sources are orthogonal to the noise subspace. As the noise subspace is orthogonal to the signal subspace, these steering vectors are contained in the signal subspace.

It should be noted that the noise subspace is spanned by the eigenvectors associated with the smaller eigenvalues of the correlation matrix, and the signal subspace is spanned by the eigenvectors associated with its larger eigenvalues.

In principle, the eigenstructure-based methods search for directions such that the steering vectors associated with these directions are orthogonal to the noise subspace and are contained in the signal subspace. In practice, the search may be divided into two parts. First, find a weight vector that is contained in the noise subspace or is orthogonal to the signal subspace. Then search for directions such that the steering vectors associated with these directions are orthogonal to this vector. The source directions correspond to the local minimas of the function $\left|\underline{w}^H \underline{s}_\theta\right|$. In this function, $\underline{s}_\theta$ denotes a steering vector.

When these steering vectors are not guaranteed to be in the signal subspace, there may be more minimas than the number of sources, and the distinction between the actual source direction and a spurious minimas in $\left|\underline{w}^H \underline{s}_\theta\right|$ is made by measuring the power in these directions.

Many methods have been proposed that utilize the eigenstructure of the array correlation matrix. These methods differ in the way the available array signals have been utilized, required array geometry, applicable signal model, and so on. Some of these methods do not require explicit computation of the eigenvalues and eigenvectors of the array correlation matrix, whereas in others, it is essential. An effective computation of these quantities may be made by methods similar to those described in [311]. When this matrix is not available, a suitable estimate of the matrix is made from the available samples.

One of the earliest methods of DOA estimation based on the eigenstructure of a covariance matrix is due to Pisarenko [312] and has a better resolution property than those of the minimum variance, maximum entropy, and linear prediction methods [313]. A critical comparison of this method with two other schemes [314], [315] applicable for a correlated noise field that exists in situations of multipaths has been presented in [316] to show that Pisarenko's method is an economized version of these schemes restricted to equispaced linear arrays. The scheme presented in [314] is useful for off-line implementation similar to those presented in [16], [317], and [318], whereas the method described in [315] is useful for real-time implementations and uses a normalized gradient algorithm to estimate a vector in the noise subspace from available array signals. Some other schemes suitable for real-time implementation are discussed in [319]–[321]. A scheme known as the matrix pencil method, shown [322] to be similar to Pisarenko's method, has been described in [323].

Eigenstructure methods may also be used for finding DOA when the background noise is not white but has either a known covariance [324] or an unknown covariance [325], or when the sources are in the near field and/or the sensors have unknown gain patterns [326]. For the latter case, the signals induced on all elements of the array are not of equal intensity, as is the case when the array is in the far field of the directional sources. The effect of spatial coherence

on the resolution capability of these methods is discussed in [327] and [328], whereas the issue of the optimality of these methods is considered in [329]. Now, some of the popular schemes are described in detail.

G. MUSIC Algorithm

1) Spectral MUSIC: The MUSIC method [330] is a relatively simple and efficient eigenstructure method of DOA estimation. It has many variations and is perhaps the most studied method in its class. In its standard form, also known as spectral MUSIC, the method estimates the noise subspace from the available samples. This can be done by either eigenvalue decomposition of the estimated array correlation matrix or singular value decomposition of the data matrix, with its N columns being the N snapshots or the array signal vectors. The latter is preferred for numerical reasons [331].

Once the noise subspace has been estimated, a search for M directions is made by looking for steering vectors that are as orthogonal to the noise subspace as possible. This is normally accomplished by searching for peaks in the MUSIC spectrum given by

$$P_{\mathrm{MU}}(\theta) = \frac{1}{\left|\underline{s}_\theta^H U_N\right|^2} \tag{163}$$

where U_N denotes an L by $L{-}M$ dimensional matrix with its $L{-}M$ columns being the eigenvectors corresponding to the $L{-}M$ smallest eigenvalues of the array correlation matrix, and $\underline{s}_\theta$ denotes the steering vector corresponding to direction θ.

It should be noted that instead of using the noise subspace and searching for directions with steering vectors orthogonal to this subspace, one may use the signal subspace and search for directions with steering vectors contained in this space [332]. This amounts to searching for peaks in $|U_S^H \underline{s}_\theta|^2$ where U_S denotes an L by M-dimensional matrix, with its M columns being the eigenvectors corresponding to the M largest eigenvalues of the array correlation matrix. It is advantageous to use the one with the smaller dimensions.

For the case of a single source, the DOA estimate made by the MUSIC method asymptotically approaches the CRLB, that is, when the number of snapshots increases infinitely, the best possible estimate is made. For the multiple sources, the same holds for the large SNR cases, that is, when the SNR approaches infinity [333], [334]. The CRLB gives the theoretically lowest value of the covariance of an unbiased estimator.

An application of the MUSIC algorithm to cellular mobile communications is investigated to locate land mobiles and shows that when multipath arrivals are grouped in clusters, the algorithm is able to locate the mean of each cluster arriving at a mobile [335]. This information then may be used to locate the line of sight. Its use for mobile satellite communications has been suggested in [59].

2) Root-MUSIC: For a ULA, the search for DOA can be made by finding the roots of a polynomial. In this case, the method is known as root-MUSIC [332]. Thus, root-MUSIC is applicable when a ULA is used. It solves a polynomial rooting problem in contrast to the identification and localization of spectral peaks using spectral MUSIC. Root-MUSIC has a better performance than spectral MUSIC [336].

3) Constrained MUSIC: This incorporates the knowledge of the known source to improve the estimates of the unknown source direction [331]. The situation arises when some of the source directions are already known. This method removes the components of the signal induced by these known sources from the data matrix and then uses the modified data matrix for DOA estimation. It is achieved by projecting the data matrix onto a space-orthogonal complement to a space spanned by the steering vectors associated with known source directions. It is a matrix operation. The process reduces the dimension of the signal subspace by a number equal to the known sources and improves the quality of the estimate, particularly when the known sources are strong or correlated with the unknown sources.

4) Beam-Space MUSIC: The MUSIC algorithms described above process the snapshots received from sensor elements without any preprocessing, such as to form beams, and thus may be thought of as element-space algorithms. This is contrary to a beam-space MUSIC algorithm, where the array data are passed through a beam-forming processor before applying MUSIC or any other DOA estimation algorithm. The output of the beam-forming processor may be thought of as a set of beams, and thus the processing using these data is normally referred to as beam-space processing. A number of DOA estimation schemes are discussed in [337] and [338], where data are obtained by forming multiple beams using an array.

DOA estimation in beam space has a number of advantages, such as reduced computation, improved resolution, reduced sensitivity to system errors, reduced resolution threshold, reduced bias in the estimate, and so on [333], [339]–[342]. These advantages arise from the fact that a beam former is used to form a number of beams that are less than the number of elements in the array, and thus one needs to process less data for DOA estimation.

One may think of this process in terms of the degrees of freedom of the array. The element-space methods have degrees of freedom equal to the number of elements in the array, whereas the degrees of freedom of beam-space methods equal the number of beams formed by the beam-forming filter. Thus, the process reduces the degrees of freedom of the array. Normally, one needs only $M + 1$ degrees of freedom to resolve M sources.

The root-MUSIC algorithm discussed for the element-space case may also be applied to this case, giving rise to beam-space root-MUSIC [341], [342]. It enjoys the computational savings offered by beam-space methods compared to element-space methods in general.

H. Min-Norm Method

The min-norm method [314], [343] is applicable for ULA and finds the DOA estimate by searching for the location

of peaks in the spectrum [344]

$$P_{\text{MN}}(\theta) = \frac{1}{|\underline{w}^H \underline{s}_\theta|^2} \qquad (164)$$

by calculating an array weight $\underline{w}$, which is of minimum norm, has its first element equal to unity, and is contained in the noise subspace. The solution of the above problem leads to the following expression for the spectrum [344]–[346]

$$P_{\text{MN}}(\theta) = \frac{1}{\left|\underline{s}_\theta^H U_N U_N^H \underline{e}_1\right|^2} \qquad (165)$$

with the vector $\underline{e}_1$ denoting all zeros except the first element, which is equal to unity. As the method is applicable for ULA, the optimization problem to solve for the array weight may be transformed to a polynomial rooting problem, leading to a root-min-norm method similar to root-MUSIC. A comparison of the performance of the two [347] indicates that the variance in the estimate obtained by root-MUSIC is smaller than or equal to that of the root-min-norm method. Schemes to speed up the DOA estimation algorithm of min-norm and to reduce computations are discussed in [344] and [348].

I. CLOSEST Method

This method is useful for locating sources in a selected sector. Contrary to beam-space methods, which work by first forming beams in selected directions, it operates in the element space and in that sense is an alternative to beam-space MUSIC. In a way, it is a generalization of the min-norm method. It searches for array weights in the noise subspace that are close to the steering vectors corresponding to the DOA's in the sector under consideration; thus the name "CLOSEST" method. Depending upon the definition of the closeness, it leads to various schemes. A method referred to as FINE selects an array weight vector by minimizing the angle between the selected vector and the subspace spanned by the steering vectors corresponding to the DOA's in the selected sector. In short, the method replaces the vector $\underline{e}_1$ used in the min-norm method with a suitable vector depending upon the definition of the closeness used. More details about the selection of these vectors and the relative merits of the CLOSEST method are provided in [349].

A number of eigenstructure methods reported in the literature exploit specialized array structures or noise scenarios. Two methods using uniform circular arrays are presented in [350] that extend beam-space MUSIC and ESPRIT algorithms (to be discussed in Section IV-J) for two-dimensional angle estimation, including an analysis of MUSIC to resolve two sources in the presence of gain, phase, and location errors. Properties of the array have also been exploited in [351] to find the azimuth and the elevation of a directional source. Two DOA estimation schemes in an unknown noise field using two separate arrays proposed in [352] appear to offer a superior performance compared to their conventional counterparts.

Advantages of minimum redundancy linear arrays are discussed in [341]. It has been shown that by using such arrays, one may be able to resolve more than $L(L-1)/2$ sources using L elements. The other direction-finding methods applicable to an unknown noise field are described in [325] and [353]–[356]. The MAP method presented in [354] and [355] is based on Bayesian analysis, and estimated results are not asystematically consistent, that is, the results may be biased [352]. The method in [356], referred to as CANAL, may be implemented using analog hardware, thus eliminating the need for sampling, data storage, and so on. A DOA estimation method in the presence of correlated arrivals using an array of unrestricted geometry is discussed in [357].

J. ESPRIT

ESPRIT [358] is a computationally efficient and robust method of DOA estimation. It uses two identical arrays in the sense that array elements need to form matched pairs with an identical displacement vector, that is, the second element of each pair ought to be displaced by the same distance and in the same direction relative to the first element.

This, however, does not mean that one has to have two separate arrays. The array geometry should be such that the elements could be selected to have this property. For example, a ULA of four identical elements with an interelement spacing d may be thought of as two arrays of three matched pairs, one with the first three elements and one with the last three elements such that the first and second elements form one pair, the second and third elements form another pair, and so on. The two arrays are displaced by the distance d. The way that ESPRIT [358] exploits this subarray structure for DOA estimation is now briefly described.

Let the signals induced on the ℓth pair due to a narrowband source in direction θ be denoted by $x_\ell(t)$ and $y_\ell(t)$. The phase difference between these two signals depends upon the time taken by the plane wave arriving from the source under consideration to travel from one element to the other. As the two elements are separated by the displacement Δ_0, it follows that

$$y_\ell(t) = x_\ell(t)e^{j2\pi\Delta_0 \cos\theta} \qquad (166)$$

where Δ_0 is measured in wavelengths. Note that Δ_0 is the magnitude of the displacement vector. This vector sets the reference direction, and all angles are measured with reference to this vector. Let the array signals received by the two arrays be denoted by $\underline{x}(t)$ and $\underline{y}(t)$. These are given by

$$\underline{x}(t) = A\underline{s}(t) + \underline{n}_x(t) \qquad (167)$$

and

$$\underline{y}(t) = A\Phi\underline{s}(t) + \underline{n}_y(t) \qquad (168)$$

where A is a K by M matrix, with its columns denoting the M steering vectors corresponding to M directional sources associated with the first subarray; Φ is an M by M diagonal matrix, with its mth diagonal element given by

$$\Phi_{mm} = e^{j2\pi\Delta_0 \cos\theta_m} \qquad (169)$$

$\underline{s}(t)$ denotes M source signals induced on a reference element; and $\underline{n}_x(t)$ and $\underline{n}_y(t)$, respectively, denote the noise induced on the elements of the two subarrays. Comparing the equations for $\underline{x}(t)$ and $\underline{y}(t)$, it follows that the steering vectors corresponding to M directional sources associated with the second subarray are given by $A\Phi$.

Let U_x and U_y denote two K by M matrixes with their columns denoting the M eigenvectors corresponding to the largest eigenvalues of the two array correlation matrixes R_{xx} and R_{yy}, respectively. As these two sets of eigenvectors span the same M-dimensional signal space, it follows that these two matrixes U_x and U_y are related by a unique nonsingular transformation matrix ψ, that is

$$U_x\psi = U_y. \tag{170}$$

Similarly, these matrixes are related to steering vector matrixes A and $A\Phi$ by another unique nonsingular transformation matrix T, as the same signal subspace is spanned by these steering vectors. Thus

$$U_x = AT \tag{171}$$

and

$$U_y = A\Phi T. \tag{172}$$

Substituting for U_x and U_y and the fact that A is of full rank, one obtains

$$T\psi T^{-1} = \Phi \tag{173}$$

which states that the eigenvalues of ψ are equal to the diagonal elements of Φ and that the columns of T are eigenvectors of ψ. This is the main relationship in the development of ESPRIT [358]. It requires an estimate of ψ from the measurement $\underline{x}(t)$ and $\underline{y}(t)$. An eigendecomposition of ψ gives its eigenvalues, and by equating them to Φ leads to the DOA estimates

$$\theta_m = \cos^{-1}\left\{\frac{\mathrm{Arg}(\lambda_m)}{2\pi\Delta_0}\right\}, \qquad m = 1, \cdots, M. \tag{174}$$

How one obtains an estimate of ψ from the array signal measurements efficiently has led to many versions of ESPRIT [358]–[363]. One version, refered to as TLS ESPRIT [358], [359], is summarized below.

1) Make measurements from two identical subarrays, which are displaced by Δ_0. Estimate the two array correlation matrixes from the measurements and find their eigenvalues and eigenvectors.

2) Find the number of directional sources M using available methods (some are described later in this section).

3) Form the two matrixes with their columns being the M eigenvectors associated with the largest eigenvalues of each correlation matrix. Let these be denoted by U_x and U_y. For a ULA, this could be done by first forming an L by M matrix U by selecting its columns as the M eigenvectors associated with the largest eigenvalues of the estimated array correlation

matrix of the full array of L elements. Then select the first $K < L$ rows of U to form U_x and the last of its K rows to form U_y.

4) Form a $2M$ by $2M$ matrix

$$\begin{bmatrix} U_x^H \\ U_y^H \end{bmatrix}[U_x \quad U_y] \tag{175}$$

and find its eigenvectors. Let these eigenvectors be the columns of a matrix V.

5) Partition V into $4M$ by M matrixes as

$$V = \begin{pmatrix} V_{11} & V_{12} \\ V_{21} & V_{22} \end{pmatrix}. \tag{176}$$

6) Calculate the eigenvalues $\lambda_m, m = 1, \cdots, M$ of the matrix

$$-V_{11}V_{22}^{-1}.$$

7) Estimate the angle of arrival θ_m using

$$\theta_m = \cos^{-1}\left\{\frac{\mathrm{Arg}(\lambda_m)}{2\pi\Delta_0}\right\}, \qquad m = 1, \cdots, M. \tag{177}$$

Other ESPRIT variations include beam-space ESPRIT [359], beam-space ESPRIT for uniform rectangular array [364], resolution-enhanced ESPRIT [360], virtual interpolated array ESPRIT [362], multiple invariance ESPRIT [365], higher order ESPRIT [366], and procrustes rotation-based ESPRIT [367].

Use of ESPRIT for DOA estimation using an array at a base station in the reverse link of a mobile communications system has been studied in [368].

K. WSF Method

The WSF method [369], [370] is a unified approach to schemes like MLM, MUSIC, and ESPRIT. It requires knowledge of the number of directional sources. The method finds the DOA such that the weighted version of a matrix whose columns are the steering vectors associated with these directions is close to a data-dependent matrix. The data-dependent matrix could be a Hermitian square root of the array correlation matrix or a matrix whose columns are the eigenvectors associated with the largest eigenvalues of the array correlation matrix. The framework proposed in the method can be used for deriving common numerical algorithms for various eigenstructure methods as well as for their performance studies. Its application for mobile communications employing an array at the base station has been investigated in [58] and [371].

L. Other Methods

A number of methods that do not require eigenvalue decomposition are discussed in [372]–[379].

The method proposed in [372] is applicable for a linear array of L elements. It forms a K by K correlation matrix from one snapshot with $K \geq M$, and is based on the QR orthonormal decomposition [380] on this correlation matrix, with Q being a K by K unitary matrix and R being an

upper triangle. The last K–M column of Q defines a set of orthonormal basis for the noise space. Denoting these columns by U_N, the directions of sources are obtained from the peaks of the spectra

$$P(\theta) = \frac{1}{\left| \underline{s}_\theta^H U_N \right|^2}. \tag{178}$$

This method is computationally efficient, and its performance is comparable to that of MUSIC [372]. A multiple-source location method based on a matrix decomposition approach is presented in [373]. The method requires knowledge of the noise power estimate and is applicable for coherent as well as noncoherent arrivals. It does not require knowledge of the number of sources.

The method discussed in [374] exploits the cyclostationarity [381] of data that may exist in certain situations. This method has significant implementation advantages, and its performance is comparable with the other methods. Another method [375] that combines accuracy with a low computation requirement using polynomial rooting exploits diversity polarization of the arrays. These arrays have the capability of separating signals based on the polarization characteristics and thus have an advantage over uniformly polarized arrays [382], [383]. An adaptive scheme based on Kalman filtering to estimate the noise subspace is presented in [377], which then is combined with root-MUSIC to estimate DOA. The method has good convergence characteristics. The method presented in [376] uses a deconvolution approach to the output of a conventional processor to a localized source, whereas those discussed in [378] and [379] use a neural-network approach to direction finding.

The discussion on DOA estimation thus far has concentrated on estimating the directions of stationary narrow-band sources. Though an extension of a narrow-band direction-finding scheme to the broad-band case is not trivial, some of the methods discussed here have been extended to estimate the directions of broad-band sources. A discussion of these and other schemes is contained in [313] and [384]–[394]. The methods described in [384]–[386], [389], and [393] are based upon a signal subspace approach, whereas those discussed in [388], [394] and [390], [395] are related to the ESPRIT method and the ML method, respectively. The application of high-resolution direction-finding methods to estimate the directions of moving sources and to track these sources may be found in [396]–[400]. The problem of estimating the mean DOA of spatially distributed sources such as exist in base-mobile communications systems has been examined in [401] and [402].

M. Preprocessing Techniques

A number of techniques are used to process data before using direction-finding methods for DOA estimation, particularly in situations where directional sources are correlated or coherent. Correlation of directional sources may exist due to multipath propagation. It tends to reduce the rank of the array correlation matrix. A correlation matrix may be tested for source coherency by applying the rank profile test described in [403]. Most preprocessing techniques try to either restore this rank deficiency in the correlation matrix or modify it to be useful for the DOA estimation methods.

One scheme, referred to as the spatial smoothing method, has been widely studied in the literature [404]–[416] and is applicable for a linear array. In its basic form, it decorrelates the correlated arrival by subdividing the array into a number of smaller overlapping subarrays and then averaging the array correlation matrix obtained from each subarray. The number of subarrays obtained from an array depends upon the number of elements used in each subarray. For example, using K elements in each subarray, one may form $L-K+1$ subarrays from an array of L elements by forming the first subarray using elements 1 to K, the second subarray using elements 2 to $K + 1$, and so on. The number and size of the subarrays are determined from the number of directional sources under consideration. For M sources, one needs a subarray size of $M + 1$ and a number of subarrays greater than or equal to M [404].

Thus, to estimate the directions of M sources, one requires an array size of $L = 2M$, which could be reduced to $3\,M/2$ by using improved spatial smoothing methods [405], [407], also known as forward-backward spatial smoothing. This process uses the average of correlation matrix obtained from the forward subarray scheme described above, which subdivides the array starting from one side of the array, and the complex conjugate of the matrix obtained from the backward subarray method, which is starting to subdivide from the other side of the array. The mth subarray matrix $\bar{R}_m$ of the backward method is related to the forward method matrix R_m by

$$\bar{R}_m = J_0\,R_m\,J_0 \tag{179}$$

where J_0 is a reflection matrix, with all its elements along the secondary diagonal equal to unity and elsewhere equal to zero. The process is similar to that used by forward-backward prediction for bearing estimation [408].

An improved spatial smoothing method [410] uses correlation between all elements of the array rather than correlation between elements of subarrays, as is normally done to improve the performance of the spatial smoothing method. The method described in [409] and [411] removes the effects of sensor noise to make spatial smoothing more effective in low-SNR situations. This spatial filtering method is further refined in [417] to offer DOA estimates of coherent sources with reduced RMS errors.

A decorrelation analysis of spatial smoothing [412] shows that there exists an upper bound on the number of subarrays and that the maximum distance between the subarrays depends upon the fractional bandwidth of the signals. A comprehensive analysis [413] of the use of spatial smoothing as a preprocessing technique to weighted ESPRIT and MUSIC methods of DOA estimation shows how their performance could be improved by the proper

choice of the number of subarrays and weighting matrixes. An application of ESPRIT to estimate source directions and polarization shows the improvement in its performance in the presence of coherent arrivals when it is combined with the spatial smoothing method [418].

The spatial smoothing methods using subarray arrangements reduce the effective aperture of the array as well as the degree of freedom, and thus one needs a higher number of elements to process correlated arrivals than otherwise required. The schemes that do not reduce the effective size of the array include those that restore the structure of the array correlation matrix for a linear array to that when there is no correlation. These are referred to as structured methods [419], [420]. For a linear equispaced array, the correlation matrix in the absence of correlated arrivals has a Toeplitz structure, that is, the elements of the matrix along its diagonals are equal. The correlation between sources destroys this structure. In [419], this is restored by averaging the matrix obtained in the presence of correlated arrivals by simple averaging along the diagonals, while in [420], a weighted average is used. A method using the array correlation matrix structured by averaging along its diagonals of DOA estimation discussed in [421] appears to offer computational advantages over similar methods.

Some other preprocessing schemes to decorrelate the correlated sources include random permutation [414], mechanical movement using a circular disk [422], construction of a preprocessing matrix using approximate knowledge of a DOA estimate [423], signal subspace transformation in the spatial domain [424], unitary transformation method [425], and methods based on aperture interpolations [415], [426], [427].

N. Estimating the Number of Sources

Many of the high-resolution direction-finding methods require the number of directional sources, and their performance is dependent on the perfect knowledge of these numbers. Some methods for estimating the number of these sources are discussed here.

The method most commonly referred to for detecting the number of sources was first introduced in [428] based on AIC [429] and Rissanen's MDL [430] principle. The method was further analyzed in [431] and [432] and modified in [433] and [434]. A variation of the method that is applicable to coherent sources is discussed in [325], [435], and [436]. Briefly, the method works as follows [428], [432].

1) Estimate the array correlation matrix from N independent and identically distributed samples.
2) Find the L eigenvalues λ_i, $i = 1, L$ of the correlation matrix such that $\lambda_1 > \lambda_2 > \cdots > \lambda_L$.
3) Estimate the number of sources M by solving

$$\underset{M}{\text{minimize}} \ N(L - M) \log\left(\frac{f_1(M)}{f_2(M)}\right) + f_3(M, N)$$

(180)

where

$$f_1(M) \triangleq \frac{1}{L - M} \sum_{i=M+1}^{L} \lambda_i \tag{181}$$

$$f_2(M) \triangleq \left(\prod_{i=M+1}^{L} \lambda_i\right)^{\frac{1}{L-M}} \tag{182}$$

and the penalty function

$$f_3(M, K) = \begin{cases} M(2L - M) & \text{for AIC} \\ \frac{1}{2}M(2L - M)\log N, & \text{for MDL} \end{cases}$$

(183)

with L denoting the number of elements in the array.

A modification of the method based on the MDL principle applicable to coherent sources is discussed in [435] and is further refined in [325] and [436] to improve the performance. A parametric method that does not require knowledge of the eigenvalues of the array correlation matrix is discussed in [437]. It has a better performance than some of the other methods discussed and is computationally more complex.

All methods that partition the eigenvalues of the array correlation matrix rely on the fact that the M eigenvalues corresponding to M directional sources are larger than the rest of the L–M eigenvalues corresponding to the background noise and select the threshold differently. One of the earliest methods [438] used a hypothesis-testing procedure based upon the confidence interval of noise eigenvalues, and the assignment of the threshold was subjective. A method referred to as an eigenthreshold method [439] uses a one-step prediction of the threshold for differentiating the smallest eigenvalues from the others. The method has a better performance than AIC and MDL. It has a threshold at a lower value of SNR than that of MDL and has a lower error rate than that of AIC at high SNR [439].

An alternate scheme for estimating the number of sources present uses the eigenvectors of the array correlation matrix, unlike other methods, which use the eigenvalues, and is discussed in [440]. The method is referred to as the eigenvector detection technique. It is applicable to a cluster of sources whose approximate directions are known and is able to estimate the number of sources at a lower SNR than that of AIC and MDL.

In practice, the number of sources an array may be able to resolve depends not only on the number of elements in the array but also on the array geometry, available number of snapshots, and spatial distribution of sources. Discussion on these and other issues related to the capabilities of an array uniquely to resolve a number of sources may be found in [441]–[443] and the references therein.

O. Performance Comparison

Performance analysis of direction-finding schemes has been carried out by many researchers [317], [336], [339], [340], [444]–[462]. The performance measures considered for analysis include bias, variance, resolution, CRLB, and probability of resolution. Among the most studied [339],

[340], [444]–[454] direction-finding schemes is MUSIC. Most of these studies concentrate on performance and performance comparison with other methods when a finite number of samples are used for direction finding rather than their ensemble average.

An asymptotic analysis of MUSIC with forward/backward spatial smoothing in the presence of correlated arrival shows [444] that to estimate two angles of arrival of equal power under identical conditions requires more snapshots for correlated sources than for uncorrelated sources [454]. A rigorous bias analysis of MUSIC shows [447] that estimates are biased. For a linear array in the presence of a single source, the bias increases as the source moves away from broadside. Interestingly, the bias also increases as the number of elements in the array are increased, keeping the aperture fixed.

Bias and STD are complicated functions of the array geometry, SNR, and number and directions of sources, and vary in a way inversely proportional to the number of snapshots. A poorer estimate generally results, using a lesser number of snapshots and sources with lower SNR. It is shown in [340] and [447] that the performance of conventional MUSIC is poor in the presence of correlated arrivals and fails to resolve coherent sources. Even though bias and STD both play important roles in direction estimation, the effect of bias near the threshold region is critical. A comparison of the performance of MUSIC with those of min-norm and FINE for a finite sample case [448] shows that in the low-SNR range, the min-norm estimates have the largest STD, and the MUSIC estimates have the largest bias. As these results are dependent on the SNR of the source, the performance of all three approaches the same limit as the SNR is increased. The overall performance of FINE is better than the other two in the absence of correlated arrivals.

The estimates obtained by the MUSIC and ML methods are compared with CRLB in [445] and [446] for a large sample case. The CRLB gives the theoretically lowest value of the covariance of an unbiased estimator. It decreases with the number of samples, number of sensors in the array, and SNR's of the sources [445]. The study concludes that the MUSIC estimates are the large sample realization of the ML estimates in the presence of uncorrelated arrivals. Furthermore, it shows that the variance of the MUSIC estimate is more than that of the ML estimate, and the two approach each other as the number of elements and the number of snapshots increases. Thus, using an array with a large number of elements and a large number of samples, one is able to make excellent estimates of directions of uncorrelated sources with large SNR using the MUSIC method [445]. It should be noted that the estimates of the ML method are unbiased [460]. An unbiased estimate is referred to as a consistent estimate.

An improvement in the MUSIC DOA estimation is possible by using beam-space MUSIC [339], [340]. By properly selecting a beam-forming matrix and then using the MUSIC scheme to estimate DOA, one is able to reduce the threshold level of the required SNR to resolve the closely spaced sources [339]. Though the variance of this estimate is not much different from the element-space case, it has less bias [340]. The resolution threshold of beam-space MUSIC is lower than that of the conventional min-norm method. For two closely spaced sources, however, beam-space MUSIC and beam-space min-norm provide identical performances when suitable beam-forming matrixes are selected [339].

It is shown in [453] that when beam-forming weights have conjugate symmetry (useful only for arrays with particular symmetry), beam-space MUSIC has a decorrelation property similar to backward/forward smoothing. Thus, it is useful for correlated arrival-source estimation and offers performance advantages in terms of lower variance for the estimated angle.

The resolution property of MUSIC is further analyzed in [449]–[452] and [454], which show how it depends upon the SNR, number of snapshots, array geometry, and separation angle of the two sources. Analytical expressions of probability of resolution and its variation as a function of various parameters [452] could enable one to predict the behavior of the MUSIC estimate for a given scenario. The two closely spaced sources are said to be resolved when two peaks appear in the spectrum in the vicinity of the source directions.

A comparison of the performances of MUSIC and other eigenvector methods, which use the noise eigenvectors divided by the corresponding eigenvalues for DOA estimation, indicates [317] that the performance of the former is more sensitive to the choice of an assumed number of sources compared to the actual number of sources.

A performance analysis of many versions of ESPRIT is considered in [336] and [456]–[458] and compared with other methods. Estimates obtained by subspace rotation methods, which include TAM and ESPRIT, have larger variance than those obtained by MUSIC using a large number of samples [456]. Estimates by ESPRIT using a uniform circular array are asymptotically unbiased [458]. LS-ESPRIT and TAM estimates are statistically equivalent. LS-ESPRIT and TLS-ESPRIT have the same MSE [336]. Their performance depends upon how the subarrays are selected [457]. The min-norm method is equivalent to TLS-ESPRIT [463], and root-MUSIC outperforms ESPRIT [464]. TAM is based on the state-space model and finds a DOA estimate from signal subspace. In spirit, its approach is similar to ESPRIT [336]. For Gaussian signals, the WSF method and ML method are efficient, as both attain CRLB asymptotically [455], [459]. A method is said to be efficient when it achieves CRLB.

The correlation between the sources affects the capabilities of various DOA estimation algorithms differently [465]. A study [461] of the effect of the correlation between two sources on the accuracy of DOA-finding schemes shows that the phase of the correlation is more significant than the correlation magnitude. Most of the performance analysis discussed so far assumes that the background noise is white. When this is not the case, the DOA schemes perform differently. In the presence of colored background noise, the performance of MUSIC is better than that of ESPRIT

and the min-norm method over a wide range of SNR. The performance of the min-norm method is worse than those of the other two [466].

P. Sensitivity Analysis

A sensitivity analysis of MUSIC to various perturbations is presented in [467]–[472]. A compact expression for the error covariance of the MUSIC estimates given in [467] may be used to evaluate the effect of various perturbation parameters, including gain and phase errors, effect of mutual coupling channel errors, and random perturbation in sensor locations. It should be noted that the MUSIC estimate of DOA requires knowledge of the number of sources similar to some other methods, and underestimation of source number may lead to an inaccurate estimate of DOA's [468]. A variance expression for a DOA estimate for this case has been provided in [468].

An analysis of the effect of model errors on the MUSIC resolution threshold [333], [470] and on the waveforms estimated using MUSIC [469] indicates that the probability of resolution decreases [470] with the error variance and that the sensitivity to phase errors depends more upon array aperture than on the number of elements [469] in a linear array. The effect of gain and phase error on the MSE of the MUSIC estimate of a general array is analyzed in [473]. The problem of estimating gain and phase errors of sensors whose locations are known is considered in [471].

An analysis [472] of ESPRIT under random sensor uncertainties suggests that the MUSIC estimates generally give lower MSE than ESPRIT. The former is more sensitive to both sensor gain and phase errors, whereas the latter depends only on phase errors. The study further suggests that for a linear array with a large number of elements, the MSE of the ESPRIT estimate with maximum overlapping subarrays is lower than that with nonoverlapping subarrays.

The effect of gain and phase errors on weighted eigenspace methods, including MUSIC, min-norm, FINE, and CLOSEST, is studied in [474] by deriving bias and variance expressions. It indicates that the effect is gradual up to a point, and then the increase in error magnitude causes an abrupt deterioration in the bias and variance of the estimate. The weighted methods differ from the standard ones such that a weighting matrix is used in the estimate, and that matrix could be optimized to improve the quality of the estimate under particular perturbation conditions.

The effect of nonlinearity in the system, such as the hard clipping common in digital beam formers, on spectral estimation methods in general is analyzed in [475], which shows that such distortion may be eliminated by additional preprocessing.

The effect of various perturbation methods on spectral estimation methods emphasizes the importance of a precise knowledge of various array parameters. There are various techniques to calibrate arrays, some of which are discussed in [476] and [477] and the references therein. There are schemes such as that discussed in [478] to estimate the steering vector and, in turn, the DOA from uncalibrated arrays and in [479] to estimate DOA. Discussions on

Table 1 Performance Summary of Bartlett Method

Name of method	Property	Comments, Comparison and References
• Bartlett Method	• Bias	• Biased [465] • Bartlett > LP > MLM
	• Resolution	• Depends upon array aperture
	• Sensitivity	• Robust to element position errors [280]
	• Array	• General Array

Table 2 Performance Summary of MVDR Method

Name of method	Property	Comments, Comparison and References
• MVDR Method	• Bias	• Unbiased
	• Variance	• Minimum
	• Resolution	• MVDR > Bartlett [42, 279] • Does not have best resolution of any method [280]
	• Array	• General Array

Table 3 Performance Summary of MEM

Name of method	Property	Comments, Comparison and References
• Maximum Entropy Method	• Bias	• Biased [465]
	• Resolution	• ME > MVDR > Bartlett [279] • Can resolve at lower SNR than Bartlett [42]

robustness issues of direction-finding algorithms may be found in [480] and [481].

A summary of the performance and sensitivity comparison of various DOA estimation schemes is provided in Tables 1–12.

V. EFFECT OF ERRORS

The communications system using an array of antenna elements considered so far is assumed to be free from errors and perturbations, and the results on various beam-forming schemes, adaptive algorithms, and DOA methods are based upon ideal error-free conditions. In real systems, these idealistic situations are hardly met, and the system performance is affected by the amount that the various system parameters deviate from the assumed conditions. Some of these deviations are discussed in this section.

A. Correlated Arrivals

The interference-canceling capabilities of the optimal beam formers discussed earlier assume that the signal and

Table 4 Performance Summary of Linear Prediction Method

Name of method	Property	Comments, Comparison and References
• Linear Prediction Method	• Bias	• Biased [465]
	• Resolution	• LP > MVDR [280] • > Bartlett • > ME [465]
	• Performance	• Good in low SNR conditions • Applicable for correlated arrivals [288]

Table 5 Performance Summary of MLM

Name of method	Property	Comments, Comparison and References
• MLM Method	• Bias	• Unbiased [460] • Less than LP, Bartlett [465], MUSIC [299]
	• Variance	• Less than MUSIC for small samples [298] • Asymptotically efficient for random signals [455, 459] • not efficient for finite samples [299] • Less efficient for deterministic signals than random signals [455] • Asymp. efficient for deterministic signal using very large array [460]
	• Computation	• Intensive with large samples [299] • Same for deterministic and random signals for large arrays [460]
	• Performance	• Applicable for correlated arrivals [298] • Works with one sample [298]

Table 6 Performance Summary of Element-Space MUSIC Method

Name of method	Property	Comments, Comparison and References
• Element Space MUSIC	• Bias	• Biased [447, 358]
	• Variance	• Less than ESPRIT and TAM for large samples [456, 457], min norm [349, 448] • Close to MLM [446], CLOSEST [349], FINE [448] • Variance of weighted MUSIC is more than unweighted MUSIC [446] • Asympt. efficient for large array [445, 446]
	• Resolution	• Limited by bias [446]
	• Array	• Applicable for general array, • Increasing aperture makes it robust [333]
	• Performance	• Fails to resolve correlated sources
	• Computation	• Intensive
	• Sensitivity	• Array calibration is critical [467], sensitivity to phase error depends more on array aperture than number of elements [469], preprocessing can improve resolution [470] • Correct estimate of source number is important [468] • MSE depends upon gain and phase errors and is lower than that for ESPRIT [472] • Increase in gain and phase errors beyond certain value causes an abrupt deterioration in bias and variance [474]

Table 7 Performance Summary of Beam-Space MUSIC Method

Name of method	Property	Comments, Comparison and References
• Beam Space MUSIC	• Bias	• Less than ES MUSIC [340]
	• Variance	• larger than ES MUSIC [482]
	• RMS Error	• Less than ESPRIT, min norm [466]
	• Resolution	• Similar to BS min norm, CLOSEST [349], • Better than ES MUSIC, ES min norm [339, 340] • Threshold SNR decreases as the separation between the sources increases [454]
	• Computation	• Less than ES MUSIC
	• Sensitivity	• Robust compared to ES MUSIC

interference are uncorrelated. The correlation between the desired signal and an unwanted interference exists in situations of multipath arrivals and deliberate jamming. It affects the performance of the beam former, as discussed in [52], [419], [420], and [483]–[495], and limits the applicability of various weight estimation schemes. For example, when the weights are estimated by minimizing the mean output power subject to look-direction constraint, the beam former cancels the desired signal while maintaining the constraint. The reason this happens is that while minimizing the mean output power, the beam former adjusts the phase of the correlated interference induced on each antenna such that the power of the sum of the signal and the interference, which is correlated with the signal, is minimized, causing the signal cancellation. This is consistent with the design that the beam former minimizes the output power. The

design of the optimal weights is based upon the assumption that the signal is not correlated with the interferences.

The correlation $\delta_{xy}(f)$ between two broad-band signals $x(t)$ and $y(t)$ is defined in terms of their power spectrum [496]

$$\delta_{xy}(f) = \frac{G_{xy}(f)}{\sqrt{G_{xx}(f)G_{yy}(f)}} \qquad (184)$$

Table 8 Performance Summary of Root-MUSIC Method

Name of method	Property	Comments, Comparison and References
• Root MUSIC	• Variance	• Less than Root min norm [347], ESPRIT [457]
	• Resolution	• BS Root MUSIC has better probability of resolution than BS MUSIC [341, 342]
	• RMS Error	• Less than LS ESPRIT [464]
	• Array	• Equispaced linear array
	• Performance	• Better than spectral MUSIC [341, 464]
		• Similar to TLS ESPRIT at SNR lower than MUSIC threshold [457]
		• BS Root MUSIC is similar to ES Root MUSIC [341, 342]

Table 9 Performance Summary of Min-Norm Method

Name of method	Property	Comments, Comparison and References
• Minimum Norm Method	• Bias	• Less than MUSIC [448, 454]
	• Resolution	• Better than CLOSEST [349], ES MUSIC [349, 454]
	• Method	• Equivalent to TLS [463]

Table 10 Performance Summary of CLOSEST Method

Name of method	Property	Comments, Comparison and References
• CLOSEST Method	• Variance	• Similar to ES MUSIC [349]
	• Resolution	• Similar to BS MUSIC,
		• Better than min norm [349]
	• Performance	• Good in clustered situation [349]
	• Sensitivity	• An increase in sensor gain and phase errors beyond certain value causes an abrupt deterioration in bias and variance [474]

Table 11 Performance Summary of ESPRIT Method

Name of method	Property	Comments, Comparison and References
• ESPRIT Method	• Bias	• TLS ESPRIT unbiased [358,458]
		• LS ESPRIT biased [358]
	• RMS Error	• Less than min norm [466]
		• TLS similar to LS [336]
	• Variance	• Less than MUSIC for large samples and difference increases with number of elements in array [456]
	• Computation	• Less than MUSIC [362]
		• BS ESPRIT needs less computation than BS Root MUSIC and ES ESPRIT [359]
	• Method	• LS ESPRIT is similar to TAM [336]
	• Array	• Needs doublets, No calibration needed
	• Performance	• Optimal weighted ESPRIT is better than uniformed weighted ESPRIT [456]
		• TLS ESPRIT is better than LS ESPRIT [457]
	• Sensitivity	• Robust than MUSIC and can not handle correlated sources [457]
		• MSE robust to sensor gain errors [472]
		• MSE is lowest for maximum overlapping subarrays under sensor perturbation[472]

Table 12 Performance Summary of FINE Method

Name of method	Property	Comments, Comparison and References
• FINE Method	• Bias	• Less than MUSIC [448]
	• Resolution	• Better than MUSIC and min norm [448]
	• Variance	• Less than min norm [448]
	• Performance	• Good at low SNR

with $G_{xy}(f)$ denoting the cross-power spectrum. It is related to the cross-correlation function

$$\rho_{xy}(\tau) = E[x(t)y(t+\tau)] \qquad (185)$$

by the Fourier transform

$$G_{xy}(f) = \int_{-\infty}^{\infty} \rho_{xy}(\tau)e^{j2\pi f\tau}\,dt. \qquad (186)$$

Thus, the correlation between the signal and an interference, hereafter denoted as δ, is a complex scalar with magnitude $0 \leq |\delta| \leq 1$ and lies within the unit circuit. When the magnitude is equal to one, the two sources are said to be coherent.

The correlation between two sources affects the rank of the correlation matrix, causing it to become singular.

Rewriting the correlation matrix for the case of two correlated directional sources as

$$R = ASA^H + \sigma_n^2 I \qquad (187)$$

with the source correlation matrix given by

$$S = \begin{bmatrix} p_s & \sqrt{p_s p_i}\delta \\ \sqrt{p_s p_i}\delta^* & p_i \end{bmatrix} \qquad (188)$$

shows how the correlation between the two sources affects R. It follows from these expressions that when two sources are uncorrelated—that is, $|\delta| = 0$—S is a diagonal matrix guaranteeing R to be positive definite (assuming A is of full rank, which requires that steering vectors corresponding to all directional sources are linearly independent [35]). The presence of correlation affects the rank of S and thus of R. In the presence of correlation, the matrix R becomes ill conditioned and may not be invertible, making

it difficult to estimate the weights of the optimal beam former, which relies on the existence of the inverse of R. Thus, a beam-forming scheme that is optimal in the absence of a correlated arrival is not able to cancel a correlated interference. Many beam-forming schemes have been devised to cancel an interference source, which is correlated with the signal. In principle, these work by restoring the rank of R.

In some earlier work [52], [497], a mechanical movement of the array perpendicular to the look direction was suggested to reduce the signal-cancellation effect by the correlated interference. The scheme generally known as a spatial dither algorithm works on the principle that as the movement is perpendicular to the look direction, the signal induced in the array is not affected, whereas the interference that arrives from a direction different than that of the signal becomes modulated with this motion. This causes a reduction in the interference as noted in [492], where the dither algorithm is further developed such that a mechanical movement is not required.

The spatial smoothing scheme [416], as discussed earlier, uses the same idea of spatial averaging by subdividing the array into smaller subarrays and estimates the array correlation by averaging the correlation matrixes estimated from each such subarray. The use of spatial smoothing for beam forming is discussed in [484] and [487] and shows that the use of this method reduces effective correlation between the interference and the desired signal, resulting in a reduction in signal cancellation caused by the optimal beam forming.

The spatial smoothing method uses uniform averaging of all the matrixes obtained from different subarrays, that is, each matrix is weighted equally. This results in an estimate of the matrix that is not as good as one could have obtained from given subarray matrixes. Ideally, in the absence of correlation, the array correlation matrix for a uniformly spaced linear array has a Toeplitz structure, that is, elements of the matrix along each diagonal are equal, and the estimated matrix by the spatial smoothing scheme is not the closest to the Toeplitz matrix. This is done by a spatial averaging technique [420], [422], which weighs each subarray matrix differently and then optimizes the weights such that it minimizes the MSE between the weighted matrix and a Toeplitz matrix. The system that results from using this matrix to estimate the weights of the beam former reduces more interference than that given by the use of a uniform weighted matrix estimate.

It should be noted that the number of rows and columns in the estimated matrix is equal to the number of elements in the subarray and not equal to the number of elements in the full array. Thus, the weights estimated by this matrix could only be applied to one of the subarrays. Consequently, not all elements of the array are used for beam forming. This reduces the array aperture and its degree of freedom. For an environment consisting of M direction interferences, the desired signal the size of the subarray should be at least $M + 2$ and the number of subarrays should be at least $M(M + 1) + 1$ [420].

A scheme that does not reduce the degree of freedom of the array is described in [419]. It decorrelates the sources by structuring the correlation matrix to be Toeplitz by averaging along each diagonal and uses the resulting matrix to estimate the weights of the full array. An adaptive algorithm to estimate the weights of an array based upon this principle is presented in [498], and the concept is extended to broad-band beam forming in [499].

A beam-forming scheme [52] based upon master and slave concepts cancels the correlated arrival by the use of two channels. In one channel, the look-direction signal is blocked, and then weights are estimated by solving the constrained beam-forming problem. These weights are then used on the second channel. As the signal is not present at the time of weight estimation, the beam former does not cancel the signal. However, the process only works for one correlated interference. It is extended for a multiple correlated interference case in [486] where an array of $2M - 1$ elements is required to cancel $M - 1$ interferences.

The other schemes that require some knowledge of the interference, such as direction or the correlation matrix due to interference only, can be found in [485], [489], [491], [494], and [495].

Many of these schemes improve the array performance in the presence of correlated arrivals by treating the correlated components as interferences and canceling them by forming nulls in their directions using beam-forming techniques. These methods do not utilize the correlated components, as is done in diversity combining (discussed previously), where different components are added in some way to improve the signal level. A receiver known as the RAKE receiver [184], [500]–[502] achieves this increase in signal level for a CDMA satellite system by using a number of demodulators operating in parallel to track each component using the user code for that signal. The delay in the signal is identified by sliding the code sequence as required to obtain the maximum correlation with the received component. The signals are added at the baseband after appropriate delay and amplitude scaling. The receiver, however, does not cancel unwanted interferences by shaping the beam pattern.

B. Look Direction and Steering Vector Error

Knowledge of the look direction is used to constrain the array response in the direction of the signal such that the signal arriving from the look direction is passed through the array processor undistorted. The array weights of the optimal beam former are estimated by minimizing the mean out power subject to the look-direction constraint. The processor maximizes the output SNR by canceling all the interferences. A direction source is treated as an interference if it is not in the look direction. This shows the importance of the accuracy of the knowledge of the look direction. An error occurs when the look direction is not the same as the desired signal direction. For this case, the processor treats the desired signal source as an interference and attenuates it. The amount of attenuation depends upon the power of the signal and the amount of error [16], [42],

[90], [503]. A stronger signal is canceled more and a larger error causes more cancellation of the signal.

The solution to beam-pointing error is to make the beam broader so that when the signal is not precisely in the direction where it should be (the look direction), its cancellation does not take place. The normal methods of broadening the beams include multiple linear constraints [16], [504] and norm constraints [121]. The latter constraints prohibit the main beam from blowing out, as is the case in the presence of pointing error. In the process of canceling a source close to the point constraint in the look direction, the array response is increased in the direction opposite to the pointing error. A scheme that does not require broadening of the main beam to reduce the effect of pointing error has been reported in [505]. It makes use of direction-finding techniques combined with a reduced dimensional ML formulation to estimate the direction of the desired signal accurately. Effectiveness of the scheme in mobile communications situations has been demonstrated by computer simulations.

The study presented in [90] indicates that the beam-space processors in general are more robust to pointing errors than element-space errors. Some other schemes to remedy pointing errors may be found in [506]–[508].

The knowledge of the look direction appears in the weight calculation through the steering vector. The optimal weight calculation for constrained beam forming requires knowledge of the array correlation matrix and the steering vector in the look direction. Thus, the pointing error causes an error to occur in the steering vector, which is used for weight calculation.

The steering vector may also be erroneous due to other factors such as imperfection in the knowledge of the position of array elements, errors caused by finite word length arithmetic, and so on. The study of the effect of steering vectors has been reported in [29], [507], and [509]. An analytical study performed by modeling the error as an additive random error [29] indicates that the effect of error is severe in the SPNMI processor, that is, when the array correlation matrix, which is used to estimate the weights, contains the signal.

As the signal power increases, the performance of the processor deteriorates further due to errors. The sensitivity of the processor to the steering vector may be decreased by using a combination of the reference signal and steering vector to estimate the weights [510].

C. Element Failure and Element Position Error

Uncertainty in the position of an element of an array causes degradation in the array performance in general [511]–[515], and more so when the array beam pattern is determined by constrained beam forming. As discussed previously, the element position uncertainty causes steering vector error, leading to a lower array gain. The effect of the position uncertainty on the beam pattern is to create a background beam pattern similar to that of a single element in addition to the normal pattern of the array [515]. A general discussion of the effect of various errors on the array pattern is provided in [516].

The position of the antenna elements of an array is normally determined by a calibration process requiring auxiliary sources in known locations [517], [518]. A procedure that does not require the location of these sources is described in [508] and [519].

The element failure tends to cause an increase in side-lobe level, and the weights estimated for the full array do not remain optimal [513]. This requires recalculation of the optimal weight with the knowledge of the failed elements taken into account [513], [514].

D. Weight Errors

Array weights are calculated using ideal conditions, stored in memory, and implemented using amplifiers and phase shifters. A theoretical study of the performance of the system assumes the ideal error-free weights, whereas the actual performance of the system is dependent upon the implemented weights. The amplitude as well as the phase of these weights are different from the ideal ones, and these differences arise from many types of errors caused at various points in the system, including:

- deviation in assumption that a plane wave arrives at the array;
- uncertainty in the positions and characteristics of the elements of the array;
- error in the knowledge of the array correlation matrix caused by its estimation from a finite sample and arithmetic;
- error in the steering vector or the reference signal used to calculate weights;
- computational error caused by finite-precision arithmetic;
- quantization error in converting the analog weights into digital form for storage;
- implementation error caused by component variation.

Studies of weight errors have been conducted by modeling these errors as random fluctuations in weights [29], [520]–[524] or by modeling them as errors in the amplitude and phase [514], [525]–[529]. Performance indexes to measure the effect of errors include the array gain [29], [525], reduction in null depth [520], reduction in interference rejection capability [523], change in side-lobe level [514], [526], [527], and bias in the angle of arrival estimation [528].

The array gain is the ratio of the output SNR to the input SNR. The effect of random weight fluctuation is to cause a reduction in the array gain. The effect is sensitive to the number of elements in the array and the array gain of the error free system [29]. For an array with a large number of elements and with a large error-free gain, a large weight fluctuation could reduce its array gain to unity, implying that output SNR becomes equal to input SNR and no array gain is obtainable.

The phase of the array weight is an important parameter, and an error in the phase may cause an estimate of the source to appear in a wrong direction when an array is used for finding DOA (see, for example, [528]). The phase control of signals is used to steer the main beam of the array in desired positions, as in electronic steering. A device normally used for this purpose is a phase shifter. Those commonly available are ferrite phase shifters and diode phase shifters [20], [530]. One of the specifications with which an array designer is concerned is the RMS phase error.

Analysis of the RMS phase error shows that it causes the output SNR of the constrained optimal process to suppress the desired signal, and the suppression is proportional to the product of the signal power and the variance of the random error [531]. Furthermore, the suppression is maximum in the absence of directional interferences. An error that occurs in digital phase shifters is quantization error. In a p-bit digital phase shifter, the minimum value of the phase that can be changed equals $2\pi/2^p$. Assuming that the error is distributed uniformly between $\pi/2^p$ to $\pi/2^p$, the variance of this error equals $\pi^2/(3 \times 2^{2p})$ [531].

The effect of perturbation in the media, which causes the wavefront to deviate from the plane wave propagation assumption, and other related issues may be found in [532]–[534]. The effect of a finite number of samples used in weight estimation is considered in [535]–[537], and how bandwidth affects the performance of a narrow-band beam former is discussed in [74] and [538]. The effect of amplitude and phase errors on a mobile satellite communications system using a spherical array employing digital beam forming is studied in [171].

E. Robust Beam Forming

The perturbation of many array parameters from their ideal conditions under which the theoretical performance of the system is predicted causes degradation in the system performance by reducing the array gain and altering the beam pattern. Various schemes have been proposed to overcome these problems and to enhance the array system performance under nonideal conditions [90], [121], [539]–[546]. Many of these schemes impose various kinds of constraints on the beam pattern to alleviate the problem caused by parameter perturbation. A survey of robust signal processing techniques in general is conducted in [547]. It contains an excellent reference list and discusses various issues concerning robustness.

VI. Conclusion

This paper has dealt with many facets of array signal processing and beam forming. The emphasis has been on presenting the results in a manner suitable for nonspecialists.

This paper has introduced the concepts of beam forming and has provided details of various beam-forming schemes. Many of the available iterative schemes applicable to adaptive beam forming have been described, along with their convergence characteristics and computational requirements.

A detailed treatment of various methods of estimating the DOA's has been provided by including the description, limitation, and capability of each method and their performance comparison as well as their sensitivity to parameter perturbations.

This paper provides references to studies where array beam-forming and DOA schemes are considered for mobile communications systems. This aspect of array signal processing was dealt with in Part I of this paper in much more detail by describing how an array could be used for mobile communications and how its use could improve the performance of such systems as well as by discussing the feasibility of an array system in a mobile communications environment.

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